zbMATH Open
THE FIRST RESOURCE FOR MATHEMATICS

Conn, Jack F.

Normal forms for smooth Poisson structures. (English) Zbl 0592.58025

Ann. Math. (2) 121, 565-593 (1985).

In his latest paper [ibid. 119, 577-601 (1984; Zbl 0553.58004)], the author constructed analytic linearizing coordinates for real-analytic Poisson tensors under a certain nondegeneracy condition.

In this paper he proves the following linearizing theorem for C^∞ Poisson tensors: Theorem. Let

$$P = () \sum_{1\le i,j\le n} P_{ij}(x)\partial/\partial x_i \wedge \partial/\partial x_j, \quad with \quad P_{ji} = -P_{ij}$$

be a Poisson tensor of class C^∞ defined in a neighborhood of the origin O in R^n, with $P(O) = 0$. Let

$$P(x) = \sum_{1\le i<j\le n, 1\le k\le n} c^k_{ij} x_k \partial/\partial x_i \wedge \partial/\partial \quad x_j +$$

$$\sum_{1\le i<j\le n} R_{ij}(x)\partial/\partial x_i \wedge \partial/\partial x_j, \quad order(R_{ij}) \ge 2$$

be the Taylor expansion to order 1 of P about 0. Suppose that the real Lie algebra $\mathfrak{g}$ for which the scalars $\{c^k_{ij};\ 1 \le i < j \le n,\ 1 \le k \le n\}$ form a set of structure constants is semisimple and of compact type. Then P is smoothly linearizable; that is, there exist C^∞ coordinates $y_1, ..., y_n$ defined in a neighborhood of the origin, of the form $y_j(x) = x_j + f_j(x)$, $order(f_j) \ge 2$, in terms of which P is expressed as

$$P(y) = \sum_{1\le i<j\le n, 1\le k\le n} c^k_{ij} y_k \partial/\partial y_i \wedge \partial/\partial \quad y_j.$$

In the proof of this result, the author views the effect upon P of changes of coordinates as a nonlinear partial differential operator. A combination of Newton's method with smoothing operators, as devised by J. Nash and J. Moser, is used to construct successive approximations to the desired coordinate system.

The author obtains the following corollary. Let P be a Poisson tensor of class C^∞ on a manifold M. Suppose that $x \in M$ is a point at which the linear term of the singular part of P is semisimple and of compact type. Then every smooth infinitesimal automorphism of P defined near x is locally a Hamiltonian vector field.

Reviewer: M.Adachi

MSC:

37J99	Dynamical aspects of finite-dimensional Hamiltonian and Lagrangian systems
58C15	Implicit function theorems; global Newton methods on manifolds

Keywords:
Poisson structure

Full Text: DOI