Stanley, Richard P.

Two poset polytopes. (English) Zbl 0595.52008

Discrete Comput. Geom. 1, 9-23 (1986).

With a partially ordered set P with n elements, the author associates two n-dimensional convex polytopes, the order polytope $\mathcal{O}(P)$ and the chain polytope $\mathcal{C}(P)$. He determines the face lattice of $\mathcal{O}(P)$, the vertices of $\mathcal{C}(P)$, and describes a piecewise-linear bijection from $\mathcal{O}(P)$ onto $\mathcal{C}(P)$ which allows to transfer properties of $\mathcal{O}(P)$ over to $\mathcal{C}(P)$. It is shown that the Ehrhart polynomials of $\mathcal{O}(P)$ and $\mathcal{C}(P)$ satisfy $i(\mathcal{O}(P),m) = i(\mathcal{C}(P),m) = \Omega(P,m+1)$, where $\Omega(P,m)$, the order polynomial, is the number of order-preserving maps $P \to \{1,\dots,m\}$. In particular, $n!\operatorname{vol}\mathcal{O}(P) = n!\operatorname{vol}\mathcal{C}(P)$ is the number of linear extensions of P. Similarly as in a former paper [J. Comb. Theory Ser. A 31, 56–65 (1981; Zbl 0484.05012)], the author uses the Aleksandrov-Fenchel inequalities for mixed volumes in connection with $\mathcal{C}(P)$ to obtain new log-concave sequences involving linear extensions of P.

Reviewer: Rolf Schneider (Freiburg im Breisgau)

MSC:

52B12 Special polytopes (linear programming, centrally symmetric, etc.)
06A06 Partial orders, general

Keywords:

partial order; convex polytopes; order polytope; chain polytope

Full Text: DOI EuDML

References:

[1] Björner, A.; Garsia, A.; Stanley, R.; Rival, I. (ed.), An introduction to the theory of Cohen-Maculay partially ordered sets, 583-616, (1982), Dordrecht-Boston-London

[2] Chvátal, V., On certain polytopes associated with graphs, J. Combin. Theory Ser B, 18, 138-154, (1975) · Zbl 0277.05139

[3] L. Comtet, Advanced Combinatorics, Reidel, Dordrecht-Boston, 1974. · Zbl 0283.05001

[4] Doberkat, E. E., Problem 84-20, SIAM Review, 26, 580-580, (1984)

[5] B. Dreesen, W. Poguntke, and P. Winkler, Comparability invariance of the fixed point property, preprint. · Zbl 0577.06005

[6] L. Geissinger, A polytope associated to a finite ordered set, preprint. · Zbl 0523.06005

[7] M. C. Golumbic, Algorithmic Graph Theory and Perfect Graphs, Academic Press, New York, 1980. · Zbl 0541.05054

[8] Habib, M., Comparability invariants, Ann. Discrete Math., 23, 371-386, (1984)

[9] Kahn, J.; Saks, M., Balancing poset extensions, Order, 1, 113-126, (1984) · Zbl 0561.06004

[10] Macdonald, I. G.; Nelsen, R. B., Solution to E2701, Amer. Math. Monthly, 86, 396-396, (1979)

[11] J. S. Provan and L. J. Billera, Simplicial Complexes Associated with Convex Polyhedra, I: Constructions and Combinatorial Examples, Technical Rept. no. 402, School of Operations Research and Industrial Engineering, Cornell University, Ithaca, New York, January 1979.

[12] Stanley, R., A chromatic-like polynomial for ordered sets, 421-427, (1970), Chapel Hill

[13] R. Stanley, Ordered structures and partitions, Mem. Amer. Math. Society, no. 119, 1972. · Zbl 0246.05007

[14] Stanley, R.; Aigner, M. (ed.), Eulerian partitions of a unit hypercube, 49-49, (1977), Dordrecht-Boston

[15] Stanley, R., Decompositions of rational convex polytopes, Ann. Discrete Math., 6, 333-342, (1980) · Zbl 0812.52012

[16] Stanley, R., Two combinatorial applications of the Aleksandrov-Fenchel inequalities, J. Combin. Theory Ser A, 31, 56-65, (1981) · Zbl 0484.05012