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Logarithmic del Pezzo surfaces of rank one with non-contractible boundaries. (English)

Zbl 0596.14024
Jap. J. Math., New Ser. 10, 271-319 (1984).

In this paper and the one announced above the authors continue their study of non-complete algebraic surfaces. The first paper is a collection of technical results, which are necessary for the sequel. In the second paper the authors prove many theorems about surfaces with logarithmic Kodaira dimension $-\infty$ [for standard definitions and earlier results see *M. Miyanishi*, "Non-complete algebraic surfaces", Lect. Notes Math. 857 (1981; Zbl 0456.14018)]. - The main theorem is the following: Let (V,D) be a logarithmic del Pezzo surface of rank one with non-contractible boundary, and let $X = V - D$. Then either X is affine ruled or X is a platonic $\mathbb{A}^1_*$-fibre space.

Affine ruledness by definition is that X contains an open set isomorphic to $U \times \mathbb{A}^1$, U some curve. The "del Pezzo surface" definition is a bit long winded. Definition of platonic $\mathbb{A}^1_*$-fibre space is even more so; but over the complex numbers, the authors prove that these surfaces are nothing but quotients of the affine plane by a non-cyclic small finite subgroup of GL(2,$\mathbb{C}$) and deleting the unique singular point. - They also prove the following theorem: Let X be a non-singular rational surface with logarithmic Kodaira dimension $-\infty$. Assume that X is not affine ruled and that for a smooth completion (V,D,X) of X, the intersection matrix of D is not negative definite. Then X is an $\mathbb{A}^1_*$-fibre space over $\mathbb{P}^1$ ($\mathbb{A}^1_* = \mathbb{A}^1-$single point) (i.e., there exists a morphism $\pi : X \to \mathbb{P}^1$, surjective and the general fibre is isomorphic to $\mathbb{A}^1-\{\text{point}\}$. Moreover X is affine uniruled, i.e., there exists a dominant quasi-finite morphism $p : U \times \mathbb{A}^1 \to X$ where U is a curve.

Reviewer: N.Mohan-Kumar

MSC:

14J10	Families, moduli, classification: algebraic theory
14J25	Special surfaces

Keywords:

platonic fibre space; non-complete algebraic surfaces; surfaces with logarithmic Kodaira dimension $-\infty$; Affine ruledness