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Reidemeister torsion in knot theory. (English. Russian original) Zbl 0602.57005

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This paper studies systematically the connections between the Alexander invariants of abelian coverings of 3-manifolds and torsion invariants in the sense of Reidemeister, Franz, de Rham and Whitehead. This connection was first made by *J. Milnor* [in "A duality theorem for Reidemeister torsion", Ann. Math., II. Ser. 76, 137-147 (1962; Zbl 0108.365)].

The first section gives a comprehensive review of the most important properties of the Alexander polynomial of a link in a homology 3-sphere. It is first shown how the torsion of an abelian covering of a CW pair generalizes the notion of Alexander polynomial. (The proofs are deferred to § 2.) The major properties of Alexander polynomials are then deduced. Symmetry, augmentation, Torres conditions, satellite links, branched cyclic covers, periodic links, concordance are all treated from the point of view of torsion.

The ordinary Alexander polynomial is an element of a group ring $\mathbb{Z}[G]$ of a free abelian group, and is well defined up to multiplication by $\pm G$. Similarly the torsions of a pair (X,Y) with respect to a multiplicative homomorphism ϕ from $H_1(X)$ to the group of units of an integral domain is well defined up to multiplication by $\pm\phi(H_1(X))$. In § 3 it is shown how the notion of torsion may be refined, so as to determine the sign, once an orientation has been chosen for the real vector space $\oplus H_1(X,Y,\mathbb{R})$. In this way the sign of the Alexander polynomial may also be determined, and it becomes possible to prove for instance that the Alexander polynomial of an amphicheiral link with an even number of components is identically 0. This was first observed by *J. H. Conway* [Comput. Probl. abstract Algebra, Proc. Conf. Oxford 1967, 329-358 (1970; Zbl 0202.547)], who used his potential function. However, there has hitherto been no such "homological" proof.

In § 4 the torsion is used to give a new approach to the Conway potential function (which is, up to a change of variable, essentially a sign determined Alexander polynomial). The desirable properties of a "Conway function" are first given axiomatically, and then it is shown that a symmetrized version of the sign determined Alexander polynomial is such a function. In this way the Conway potential function may be defined for any link in a homology 3-sphere.

Sections 5 and 6 consider another modification of the torsion, defined for certain odd-dimensional Poincaré duality pairs (M,∂M) and homomorphisms from $\mathbb{Z}[H_1(M)]$ to an integral domain with involution. Although the indeterminacy in the definition of this variant is greater than that of the original torsion, it is a more useful invariant in nonacyclic contexts, for instance when the Alexander polynomial of a link is 0. The corresponding Alexander invariant is in fact the first nonzero member of the sequence of higher Alexander polynomials, and is defined up to multiplication by $\pm uf\bar{f}$, where u is in G and f is in the field of fractions of $\mathbb{Z}[G]$.

The appendix gives duality theorems for various of these notions of torsion, for manifold pairs.

Reviewer: J.Hillman

MSC:

57M25 Knots and links in the 3-sphere (MSC2010)
57Q10 Simple homotopy type, Whitehead torsion, Reidemeister-Franz torsion, etc.
57N10 Topology of general 3-manifolds (MSC2010)
57P10 Poincaré duality spaces

Keywords:

Alexander invariants of abelian coverings of 3-manifolds; torsion invariants; Reidemeister torsion; Alexander polynomial of a link; homology 3-sphere; Alexander polynomial of an amphicheiral link; Conway potential function; sign determined Alexander polynomial; Poincaré duality pairs

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