Sierksma, Gerard; **Bakker, Evert Jan**
Nearly principal minors of M-matrices. (English) Zbl 0603.15006
Compos. Math. 59, 73-79 (1986).

Let c_{ij} be the cofactor of the (i,j) element of the $n \times n$ matrix (sI-A) where $A \geq 0$, $s > 0$, and the Perron-Frobenius eigenvalue of A is $< s$. Then it is known [the first author, Linear Algebra Appl. 26, 175-201 (1979; Zbl 0409.90027)] that when $n \geq 3$, $c_{ii}c_{kj} - c_{ij}c_{ki} \geq 0$. If, more stringently, $A > 0$ and all row sums of A are strictly less than s, then $c_{kk} > c_{kj}$, $j \neq k$ ('Metzler's theorem'). Both these propositions are generalized here, the first to larger 'nearly principal' minors of the matrix $\{c_{ij}\}$. In the second the conditions on A are relaxed; unmentioned is information in the paper of *T. Fujimoto*, *C. Herrero* and *A. Villar* [ibid. 64, 85-91 (1985; Zbl 0556.15003)] including the generalization of Metzler's theorem by *M. Fiedler* and *V. Pták* [Czech. Math. J. 12, 382-400 (1962; Zbl 0131.248)].

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