Wiegmann, Klaus Werner
A theory of relative extensions for subalgebras and submodules. (English) Zbl 0605.13010
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Relative extensions of algebras and submodules are studied. These extensions appear naturally in complex analytic deformation theory. Let (A,B) and (G,f) be two pairs, where $A\subset B$ is a subalgebra over the commutative ring K, F is a B-module, $G\subset F$ is a sub-A-module. Consider all relative extensions

$$\begin{array}{ccccccccc} 0 & \to & F \to \tilde{B} & \to & B & \to 0 & & & \\ & & \cup & & \cup & & \cup & & \\ 0 & \to & G & \to & \tilde{A} & \to & A & \to & 0 \end{array}$$

such that both rows are (K-split) extensions. Let $\mathrm{Ex}(A,B;G,F)$ denote the class of all relative extensions. The equivalence relation may be established on $\mathrm{Ex}(A,B;G,F)$. In this way we obtain a set $\mathrm{Ex}(A,B;G,F)$ of equivalence classes, which endowed with two operations: $K\times\mathrm{Ex}(\cdot)\to\mathrm{Ex}(\cdot)$, $\mathrm{Ex}(\cdot)\times\mathrm{Ex}(\cdot)\to\mathrm{Ex}(\cdot)$ becomes a K-module.

Proposition 1. $\mathrm{Ex}(A,B;G,F)$ is isomorphic to the K-module

$$[C^1(A,F/G)\times_{Z^2(A,F/G)} Z^2(B,F)]/C^1(B,F).$$

The notations according to *S. MacLane* [Homology. Berlin etc.: Springer-Verlag (1963; Zbl 0133.26502); 3rd edition (1975; Zbl 0328.18009)] are used here. If K is a field, the vector space $\mathrm{Ex}(A,B;G,F)$ is isomorphic to $Z^2(A,B;G,F)/C^1(A,B;G,F)$. Using this isomorphism, the author establishes some long exact sequences of relative derivation and extension modules which interlock in an interesting way.

Reviewer: V. Sharko

MSC:

13D03 (Co)homology of commutative rings and algebras (e.g., Hochschild, André-Quillen, cyclic, dihedral, etc.)
13B02 Extension theory of commutative rings
18G15 Ext and Tor, generalizations, Künneth formula (category-theoretic aspects)

Keywords:

Relative extensions; complex analytic deformation; relative derivation

Full Text: DOI

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