Katz, Nicholas M.

On the calculation of some differential Galois groups. (English) Zbl 0609.12025

Invent. Math. 87, 13-61 (1987).

Let k be an ordinary differential field of characteristic 0 with derivation operator δ and let $L \in k[\delta]$. How can one "tell at a glance" what the Galois group of L over k is? More than twenty years ago E. Kolchin put this question at the IMC in Moscow. In spite of the separate results received in the last years by I. Kovačić, M. Singer, F. Baldassarri, V. Salikhov, the reviewer and others, there is not any hope now to receive some satisfactory answer on this question in the near future.

The author has solved the above problem in the case when $k = \mathbb{C}(x)$ and $\delta = d/dx$ for the following operators L:

(A) $L = P(\delta) + Q(x)$, where $P(\delta) \in \mathbb{C}[\delta]$, Q(x)$\in \mathbb{C}[x]$, $\deg P = n$, $\deg Q = m$, m and n are relatively prime and $n \geq 2$;

(B) $L = P(x\delta) + Q(x)$, if P and Q as above also satisfy that all roots of P are rational numbers with denominator prime to n and $Q(0) = 0$.

He proves that the Galois group of such operators is sufficiently large, i.e. it is caught between $\mathrm{SL}(n)$ and $\mathrm{GL}(n)$ or (if n is even), between $\mathrm{Sp}(n)$ and $\mathrm{GSp}(n)$. The author uses the theory of Tannakian categories and he considers that they are much better suited for discussing the problem than Kolchin's theory. The last is representing a disputable point of view for the reviewer.

Reviewer: N. V. Grigorenko (Mykola Grygorenko) (Kyïv)

MSC:

12H05	Differential algebra
34A30	Linear ordinary differential equations and systems, general
14A20	Generalizations (algebraic spaces, stacks)
14L17	Affine algebraic groups, hyperalgebra constructions
14C30	Transcendental methods, Hodge theory (algebro-geometric aspects)
14F99	(Co)homology theory in algebraic geometry

Keywords:

Kloosterman equations; affine algebraic groups; motifs; Hodge theory; differential Galois group; ordinary differential field; Tannakian categories

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