Stanley, Richard P.

**On the number of faces of centrally-symmetric simplicial polytopes.** (English) Zbl 0611.52002

Graphs Comb. 3, 55-66 (1987).

Author's abstract: "*I. Bárány* and *L. Lovász* [Acta Math. Acad. Sci. Hung. 40, 323-329 (1982; Zbl 0514.52003)] showed that a d- dimensional centrally-symmetric simplicial polytope $\mathcal{P}$ has at least $2^d$ facets, and conjectured a lower bound for the number $f_i$ of i-dimensional faces of $\mathcal{P}$ in terms of d and the number $f_0 = 2n$ of vertices. Define integers $h_0, \ldots, h_d$ by $\sum_{i=0}^{d} f_{i-1}(x-1)^{d-i} = \sum_{i=0}^{d} h_i x^{d-i}$. A. Björner conjectured (unpublished) that $h_i \geq \binom{d}{i}$ (which generalizes the result of Bárány-Lovász since $f_{d-1} = \sum h_i$), and more strongly that $h_i - h_{i-1} \geq \binom{d}{i} - \binom{d}{i-1}$, $1 \leq i \leq [d/2]$, which is easily seen to imply the conjecture of Bárány-Lovász. In this paper the conjectures of Björner are proved."

The proof uses Cohen-Macaulay simplicial complexes and toric varieties. The author points out that for the corresponding upper bound problem (largest possible value of $f_i$ for a centrally-symmetric simplicial d-polytope with $f_0 = 2n$ vertices) not even a plausible conjecture is known.

Reviewer: R.Schneider

**MSC:**

52Bxx Polytopes and polyhedra
05A20 Combinatorial inequalities



**Keywords:**

h-vector; centrally-symmetric simplicial polytope; Cohen-Macaulay simplicial complexes

**Full Text:** DOI

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