

Hida, Haruzo

Galois representations into $GL_2(\mathbb{Z}_p[[X]])$ attached to ordinary cusp forms. (English)

Zbl 0612.10021

Invent. Math. 85, 545-613 (1986).

Let p be a prime ≥ 5 , N an integer prime to p , Γ the topological group $1+p\mathbb{Z}_p$, and Ω a p -adic completion of an algebraic closure of \mathbb{Q}_p . The author uses the study of the parabolic cohomology groups of $\Gamma_1(Np^r)$ to obtain results about Galois representations attached to cusp forms. Beginning with a study of the ordinary part $h^0(N, \mathbb{Z}_p)$ of the universal Hecke algebra (generated by the Hecke operators regarded as endomorphisms of the space of p -adic cusp forms of level N) the author proves the following theorem concerning q -expansions: Let $\Lambda = \mathbb{Z}_p[[X]]$ be the one variable Iwasawa algebra, and fix a non-trivial Λ -algebra homomorphism $\lambda : h^0(N, \mathbb{Z}) \rightarrow \Lambda$. The image under λ of the formal power series $\sum_{n=1}^{\infty} T(n)q^n$ is, when evaluated at $X = \varepsilon(u)u^k - 1$ ($u = 1 + p$), the complex q -expansion of a common eigenform $f_{k,\varepsilon}$ for all $T(n)$ in $S_k(\Gamma_1(p^r))$ where $\varepsilon : \Gamma \rightarrow \Omega^*$ is of finite order, $\ker \varepsilon = 1 + p^r\mathbb{Z}_p$, and k is an integer ≥ 2 . Moreover, $f_{k,\varepsilon}$ is not the twist by any Dirichlet character of a form of lower level.

The author then shows that one can associate to λ a unique Galois representation $\pi(\lambda) : \text{Gal}(\bar{\mathbb{Q}}/\mathbb{Q}) \rightarrow GL_2(\Lambda)$ such that the reduction of $\pi(\lambda)$ is equivalent to the irreducible $\text{Gal}(\bar{\mathbb{Q}}/\mathbb{Q})$ -representation $\pi(f_{k,\varepsilon})$ into $GL_2(\Omega)$ associated to $f_{k,\varepsilon}$ (by Eichler-Shimura for $f = 2$, and by Deligne for $k > 2$).

The paper ends with a study of the connections between the special values of the L -function associated to the 3-dimensional subrepresentation contained in $\pi(f_{k,\varepsilon}) \otimes \tilde{\pi}(f_{k,\varepsilon})$ (where $\tilde{\pi}$ is the contragredient representation of π), and the characteristic power series of a certain Iwasawa module that is associated to λ .

Reviewer: [S.Kamienny](#)

MSC:

- 11F80 Galois representations
- 11F67 Special values of automorphic L -series, periods of automorphic forms, cohomology, modular symbols
- 11F33 Congruences for modular and p -adic modular forms
- 11F11 Holomorphic modular forms of integral weight
- 11F25 Hecke-Petersson operators, differential operators (one variable)
- 11R23 Iwasawa theory

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Keywords:

parabolic cohomology groups; Galois representations; cusp forms; Hecke algebra; Hecke operators; p -adic cusp forms; q -expansions; Iwasawa algebra; special values of the L -function

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