

**Gelbart, Stephen; Piatetski-Shapiro, Ilya I.; Rallis, Stephen**

**Explicit constructions of automorphic  $L$ -functions.** (English) Zbl 0612.10022

Lecture Notes in Mathematics, 1254. Berlin etc.: Springer-Verlag. vi, 152 pp. DM 28.50 (1987).

This monograph consists of two parts. The first part was written by *I. I. Piatetski-Shapiro* and *S. Rallis*. It describes a new method for giving an integral representation for a large class of  $L$ -functions associated with cuspidal automorphic representations of reductive algebraic groups. This permits one to derive the analytic properties of the  $L$ -functions in question from those of Eisenstein series associated with an auxiliary group. This generalizes the method of Godement-Jacquet; it also allows the authors to prove that the  $L$ -functions associated with a group of the form  $\mathrm{Sp}(2n)$  or a split  $\mathrm{SO}(2n)$  and the 'standard' representation of the  $L$ -group are meromorphic with only a finite set of poles.

The second part was written by *S. Gelbart* and *I. I. Piatetski-Shapiro*. It deals with the study of  $L$ -functions associated with representations of groups of the form  $G \times \mathrm{GL}(n)$  where  $G$  is a classical group of split rank  $n$ . The discussion is based on a novel integral identity, superficially the same for all classes of classical groups but with distinct proofs for each case. This again reduces the analytic properties of the  $L$ -functions to those of Eisenstein series on auxiliary groups. In this case the Eisenstein series is derived from the cuspidal representation of  $\mathrm{GL}(n)$ . The authors construct Euler products which they expect to be simple multiples of the  $L$ -functions introduced by Langlands. They also prove the functional equation for this function.

In an appendix written by all three authors the unramified factors are computed and are shown to be the expected ones.

Reviewer: [Samuel James Patterson \(Göttingen\)](#)

**MSC:**

- [11F70](#) Representation-theoretic methods; automorphic representations over local and global fields
- [11S37](#) Langlands-Weil conjectures, nonabelian class field theory
- [22E50](#) Representations of Lie and linear algebraic groups over local fields
- [11-02](#) Research exposition (monographs, survey articles) pertaining to number theory
- [22-02](#) Research exposition (monographs, survey articles) pertaining to topological groups

Cited in **6** Reviews  
Cited in **81** Documents

**Keywords:**

automorphic  $L$ -functions; integral representation; analytic properties; Eisenstein series; Euler products; functional equation; unramified factors