**Peletier, L. A.**; **Terman, D.**; **Weissler, F. B.**

**On the equation $\Delta u + x \cdot \nabla u + f(u) = 0$.** (English) Zbl 0615.35034

Arch. Ration. Mech. Anal. 94, 83-99 (1986).

The asymptotic behaviour of radial solutions of the equation

$$(1) \quad \Delta u + \frac{1}{2} x \cdot \nabla u + \frac{k}{2} u + |u|^{p-1} u = 0, \quad x \in R^N,$$

which is an ordinary differential equation

$$(2) \quad u'' + (\frac{N-1}{r} + \frac{r}{2}) u' + \frac{k}{2} u + |u|^{p-1} u = 0, \quad r = |x|, \quad u(0) = a, \quad u'(0) = 0$$

for such solutions is investigated. It is proved that if $\lim_{r\to\infty} r^k u(r) = 0$, then $u(r) = 0(e^{-\frac{r^2}{4}})$ and if $\lim_{r\to\infty} r^k u(r) \neq 0$, then $u(r) = 0(r^{-k})$. In this connection two asymptotic terms depending on the symbol of the variable (p-1)k-2 are calculated and it is shown that for $k \leq \frac{N}{2}$, $(p+1)(p-1)^{-1} \leq \frac{N}{2}$, $u(r) > 0$ for $r > 0$ and $\lim_{r\to\infty} r^k u(r) > 0$. For the solution of equation (1) and

$$\Delta u - \frac{1}{2} x \cdot \nabla u - \frac{k}{2} u + |u|^{p-1} u = 0$$

integral identities generalizing the Pokhozhaev formula were proved [*S. I. Pokhozhaev*, Dokl. Akad. Nauk SSSR 165, 36-39 (1965; Zbl 0141.302)]. The explicit formulae of solutions of equation (2) for certain combinations between the parameters N, k and p are given at the end.

Reviewer: B.Kvedaras

**MSC:**

35J60 Nonlinear elliptic equations
35B40 Asymptotic behavior of solutions to PDEs
35B30 Dependence of solutions to PDEs on initial and/or boundary data and/or on parameters of PDEs



**Keywords:**
asymptotic behaviour; radial solutions; Pokhozhaev formula; explicit formulae of solutions

**Full Text:** DOI

**References:**

[1] Brezis, H., L. A. Peletier \& D. Terman, A very singular solution of the heat equation with absorption, to appear in Arch. Rational Mech. Anal,. · Zbl 0617.35115

[2] scGidas, B., W.-M. scNi \& L. Nirenberg, Symmetry of positive solutions of nonlinear equations in ?n, Adv. in Math. Supplementary Studies (Ed. L. Nachbin) 7A (1981), 369-402.

[3] Giga, Y., \& R. V. Kohn, Asymptotically self-similar blow-up of semilinear heat equations, Comm. Pur. Appl. Math. 38 (1985), 297-319. · Zbl 0585.35051 · doi:10.1002/cpa.3160380304

[4] Haraux, A., \& F. B. Weissler, Non-uniqueness for a semilinear initial value problem, Ind. Univ. Math. J. 31 (1982), 167-189. · Zbl 0506.35055 · doi:10.1512/iumj.1982.31.31016

[5] Kamin, S., \& L. A. Peletier, Large time behaviour of solutions of the heat equation with absorption, to appear in Annali Scuola Norm. Sup. Pisa. · Zbl 0598.35050

[6] Poho?aev, S. I., Eigenfunctions of the equation ? u +? f(u) = 0, Dokl. Akad. Nauk SSSR 165 (1965), 36-39 (in Russian) and Sov. Math. 6 (1965), 1408-1411 (in English).

[7] Strauss, W. A., Existence of solitary waves in higher dimensions, Comm. Math. Phys. 55 (1977), 149-162. · Zbl 0356.35028 · doi:10.1007/BF01626517

[8] Weissler, F. B., Asymptotic analysis of an ordinary differential equation and nonuniqueness for a semilinear partial differential

equation, Archive for Rational Mech. and Anal. 91 (1986), 231-245. · Zbl 0614.35043 · doi:10.1007/BF00250743

[9] Weissler, F. B., Rapidly decaying solutions of an ordinary differential equation, with applications to semilinear elliptic and parabolic partial differential equations, in Archive for Rational Mech. and Anal. 91 (1986), 247-266. · Zbl 0604.34034 · doi:10.1007/BF00250744

[10] Atkinson, F. V., \& L. A. Peletier, Sur les solution radiales de l'équation \(\Delta u + \tfrac{1}{2}x \cdot \triangledown u + \tfrac{1}{2}\lambda u + \left| u \right|^{p - 1} u = 0\) . To appear in the C. R. Acad. Sc. Paris. · Zbl 0606.35025

[11] Escobedo, M., \& O. Kavjan, Variational problems related to self-similar solutions of the heat equation. To appear in Nonlinear Analysis, TMA.