Tahara, Hidetoshi

Singular hyperbolic systems. VI: Asymptotic analysis for Fuchsian hyperbolic equations in Gevrey classes. (English) Zbl 0621.35061

J. Math. Soc. Japan 39, 551-580 (1987).

[For Part V see ibid. 36, 449-473 (1984; Zbl 0526.35017).]

The paper deals with a class of Fuchsian hyperbolic operators of the form

$$P=(t\partial_t)^m+\sum_{j+|\alpha|\le m, j<m} t^{p_{j,\alpha}} a_{j,\alpha}(t,x)(t\partial_t)^j\partial_x^\alpha,$$

where (t,x)$\in [0,T]\times\mathbb{R}^n$, $p_{j,\alpha}\in\{0,1,2,...\}$ and $a_{j,\alpha}(t,x)\in C^\infty([0,T],\mathcal{S}^{\{s\}}(\mathbb{R}^n))$. Here, $\mathcal{E}^{\{s\}}(\mathbb{R}^n)$ denotes the set of all Gevrey functions on $\mathbb{R}^n$ of class $\{\ s\}$. Under a suitable hyperbolicity, the irregularity index $\sigma\ (\ge 1)$ is defined for P. The operator

$$L=(t\partial_t)^2-t^{2p_1}\partial_{x_1}^2-t^{2p_2}\partial_{x_2}^2+t^{q_1}a_1(t,x)\partial_{x_1}+t^{q_2}a_2(t,x)\partial_{x_2}+b(t,x)(t\partial_t)+c(t,x)$$

is a typical example, and in this case σ is given by $\sigma=\max\{1,(2p_1-q_1)/p_1,\ (2p_2-q_2)/p_2\}$. Under $1<s<\sigma/(\sigma-1)$ and a suitable assumption on the characteristic exponents of P, the following (1) and (2) are established: (1) the unique solvability of $Pu=f$ in $C^\infty([0,T],\mathcal{E}^{\{s\}}(\mathbb{R}^n))$, and (2) the asymptotic expansion (as $t\to+0$) of solutions of $Pu=0$ in $C^\infty((0,T),\mathcal{E}^{\{s\}}(\mathbb{R}^n))$. In the case $\sigma=1$, the results (1) and (2) with $\mathcal{E}^{\{s\}}(\mathbb{R}^n)$ replaced by $\mathcal{E}(\mathbb{R}^n)$ were already obtained in Part III [J. Fac. Sci., Univ. Tokyo, Sect. IA 27, 465-507 (1980; Zbl 0463.35053)] and Part V (loc. cit.).

MSC:

35L30 Initial value problems for higher-order hyperbolic equations
35C20 Asymptotic expansions of solutions to PDEs

Keywords:

Fuchsian hyperbolic operators; Gevrey functions; irregularity; unique solvability; asymptotic expansion

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