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Reduction of degenerate Lagrangian systems. (English) Zbl 0621.58020
J. Geom. Phys. 3, No. 3, 353-400 (1986).

Let a degenerate Lagrangian L be given on some velocity phase space (i.e. a tangent bundle). Is it possible to construct another velocity space and a regular Lagrangian which contains the same dynamical information as L ? This important question is referred to as the "regularization problem" for degenerate Lagrangians. The authors find conditions which guarantee the existence of such a regularization and describe a relevant class of Lagrangians for which the above question admits an affirmative answer. The connection with the canonical approach to the regularization problem of degenerate systems (Dirac's theory) and the reduction of systems with symmetry (Marsden-Weinstein theory) is also discussed. The paper is completed with a few typical examples and applications. Two appendices are devoted to the proof of those intermediate results which are of interest on their own.

Reviewer: [J.Szilasi](#)

MSC:

- [37J99](#) Dynamical aspects of finite-dimensional Hamiltonian and Lagrangian systems
- [53C15](#) General geometric structures on manifolds (almost complex, almost product structures, etc.)
- [70H03](#) Lagrange's equations
- [53C80](#) Applications of global differential geometry to the sciences

Cited in **1** Review
Cited in **29** Documents

Keywords:

[tangent bundle geometry](#); [degenerate Lagrangian](#); [regularization problem](#); [reduction](#)

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