Christensen, Erik; **Sinclair, Allan M.**

Representations of completely bounded multilinear operators. (English) Zbl 0622.46040
J. Funct. Anal. 72, 151-181 (1987).

There is a well known representation theorem for a completely bounded linear operator from a C^*-algebra into the algebra of bounded linear operators BL(H) on a Hilbert space H. This result may be proved either by employing Wittstock's decomposition of a completely bounded linear operator into a linear combination of four completely positive operators or by Arveson's extension theorem together with a dilation argument. See [*V. I. Paulsen*, Completely bounded maps and dilations, Pitman Research Notes Math., 146 (1986; Zbl 0614.47006)] for a detailed discussion of completely bounded linear operators.

In this paper the authors introduce a definition of complete boundedness for a multilinear operator from one C^*-algebra into another. Using Wittstock's matricial Hahn-Banach Theorem [*G. Wittstock*, J. Funct. Anal. 40, 127-150 (1981; Zbl 0495.46005)] they prove a corresponding representation theorem for completely bounded multilinear operators in terms of *-representations of the algebra and suitable bridging operators between the Hilbert spaces. *V. I. Paulsen* and *R. R. Smith* have recently given a proof of this result that depends on Arveson's extension theorem, and have extended the result to operator spaces [J. Funct. Anal. 73, 258-276 (1987)]. The representation theorem has been used to show that certain cohomology groups from a von Neumann algebra into an injective von Neumann algebra are zero [*E. Christensen, E. G. Effros* and *A. M. Sinclair*, Inventiones Math. 90, 279-296 (1987)].

(Correction: E. G. Effros has pointed out that K^∞ in Corollaries 5.7 and 5.8 should be replaced by $K^I = K \otimes \ell^2(I)$ for an arbitrary set I. Alternatively the conclusions are correct with the additional assumptions that the Hilbert spaces are separable and the von Neumann algebras have separable preduals).

MSC:

46L05 General theory of C^*-algebras

Keywords:

representation theorem; completely bounded linear operator from a C^*-algebra into the algebra of bounded linear operators; complete boundedness for a multilinear operator from one C^*-algebra; Wittstock's matricial Hahn-Banach Theorem; injective von Neumann algebra

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