Bousfield, A. K.

On the homology spectral sequence of a cosimplicial space. (English) Zbl 0623.55009
Am. J. Math. 109, 361-394 (1987).

A rich source of spectral sequences is the category of cosimplicial objects over a category, which is the target of the resolution associated to a triple. The construction, due to D. L. Rector for fibre squares, and more generally to D. W. Anderson, determines a spectral sequence that, when it converges, does so to the homology of the total space associated to a cosimplicial space X, Tot X= hom(Δ, X). The spectral sequence is classically in the second quadrant where the question of convergence is very delicate.

In this paper, the author determines conditions on the cosimplicial space X in order for the associated homology spectral sequence to converge strongly to $\mathcal{D}_*(TotX)$, for a spectrum $\mathcal{D}$. The proof proceeds by first reducing the question for the spectrum $\mathcal{D}$ to ordinary homology. Then by investigating the homotopy properties of the category of cosimplicial spaces, the author further reduces the problem of convergence to known results of W. G. Dwyer on the convergence of the Eilenberg-Moore spectral sequence. The arguments involving pro-objects and the new results on cosimplicial spaces are of independent interest. Among the spectral sequences to which the convergence results apply are the classical and generalized Eilenberg-Moore spectral sequences, Barratt's spectral sequence for an inclusion, and Dwyer's spectral sequence for $\mathcal{H}_*(\Omega^\infty \mathcal{W}; \mathbb{Z}/p)$.

Reviewer: J.McCleary

MSC:

55T20	Eilenberg-Moore spectral sequences
55U10	Simplicial sets and complexes in algebraic topology
18G40	Spectral sequences, hypercohomology

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convergence of spectral sequences; resolution associated to a triple; total space associated to a cosimplicial space; homology spectral sequence; Eilenberg-Moore spectral sequence; Barratt's spectral sequence; Dwyer's spectral sequence

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