Rhodes, John
Infinite iteration of matrix semigroups. II: Structure theorem for arbitrary semigroups up to aperiodic morphism. (English) Zbl 0626.20050
J. Algebra 100, 25-137 (1986).

This paper is Part II of a nearly monograph-sized study of the "global theory of semigroups" [Part I, ibid. 98, 422-451 (1986; Zbl 0584.20053)]. In part I, a structure theory for torsion semigroups was derived. It is based on an iterated matrix construction. In this second part, the structure theory for arbitrary semigroups is developed.

The "global theory of semigroups" will, given a semigroup T, determine semigroups S and X and a surjective morphism ϑ such that X is "easily globally computed", S is a "special" subsemigroup of X, and ϑ is a "fine" morphism of S onto T. In this paper, "special" means "equal", that is, $S = X$; "fine" means "aperiodic", that is, ϑ is aperiodic if and only if the pre- images of aperiodic subsemigroups of T are aperiodic subsemigroups of S; finally and roughly speaking, a semigroup X is "easily globally computed" in the context of this paper, if it is a cyclic monoid, a Rees matrix semigroup over easily globally computed semigroups with countable index sets, or an elementary projective limit of easily globally computed semigroups - at least, this is the construction for the countable case; a modified definition is also given for the uncountable case. Thus, easily globally computed semigroups in the sense of this paper are infinite iterations of matrix semigroups over cyclic monoids.

The main result of this paper is as follows: For any given monoid T there exist a monoid X and a surjective morphism $\vartheta : X \to T$ such that X is an infinite iteration of matrix semigroups over cyclic monoids and ϑ is aperiodic. The result implies that the maximal subgroups of X are finite cyclic groups.

In the course of proving this and some related results, quite a few statements of independent importance are derived. Clearly this paper is an outstanding contribution to the structure theory of semigroups and, furthermore, to the discussion concerning its methodology. The reviewer looks forward to seeing Parts III and IV.

Reviewer: H.Jürgensen

MSC:

20M10	General structure theory for semigroups
20M20	Semigroups of transformations, relations, partitions, etc.
20M15	Mappings of semigroups
20M30	Representation of semigroups; actions of semigroups on sets

Keywords:

torsion semigroups; aperiodic subsemigroups; cyclic monoid; Rees matrix semigroup; easily globally computed semigroups; infinite iterations of matrix semigroups

Full Text: DOI

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