Hitchin, Nigel
Stable bundles and integrable systems. (English) Zbl 0627.14024
Duke Math. J. 54, 91-114 (1987).

The author shows that cotangent bundles of moduli spaces of vector bundles over a Riemann surface are algebraically completely integrable Hamiltonian systems. More precisely, let G be a complex semisimple Lie group, let N be the moduli space of stable G-bundles with prescribed topological invariants on a compact Riemann surface and let n be the dimension of N. The cotangent space to N at the point represented by a G- bundle P is $H^0(M;ad(P\otimes K))$ where ad(P) is the bundle associated to P via the adjoint representation of G on its Lie algebra g. Thus a choice of basis $p_1,...,p_k$ for the ring of invariant polynomials on g induces a holomorphic map $\phi: T*N\to \oplus H^0(M;K^{d_i})$ where d_i is the degree of p_i. The components of ϕ are n functionally independent Poisson-commuting functions on T*N, and when G is a classical group the generic fibre of ϕ is an open set in an abelian variety on which the Hamiltonian vector fields defined by the components of ϕ are linear. This is what it means to say that T*N is an algebraically completely integrable Hamiltonian system. The abelian varieties occurring are either Jacobian or Prym varieties of curves covering M.

Reviewer: F.Kirwan

MSC:

14H10	Families, moduli of curves (algebraic)
14D20	Algebraic moduli problems, moduli of vector bundles
37J99	Dynamical aspects of finite-dimensional Hamiltonian and Lagrangian systems
14F05	Sheaves, derived categories of sheaves, etc. (MSC2010)
14H40	Jacobians, Prym varieties
14K10	Algebraic moduli of abelian varieties, classification

Keywords:

cotangent bundles of moduli spaces of vector bundles over a Riemann surface; completely integrable Hamiltonian systems; adjoint representation; Jacobian; Prym varieties

Full Text: DOI

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