Tanaka, Naoki
On the exponentially bounded C-semigroups. (English) Zbl 0631.47029
Tokyo J. Math. 10, No. 1-2, 107-117 (1987).

If C is an injective bounded operator with dense range on a Banach space, one defines an exponentially bounded C-semigroup as a strongly continuous family of bounded operators, S(t), $t \geq 0$, such that $S(t+s)C = S(t)S(s)$, $S(0) = C$, and $\|S(t)\| \leq Me^{at}$ [*E. B. Davies* and *M. M. N. Pang*, The Cauchy problem and a generalization of the Hille-Yosida theorem (to appear)]. The C-complete infinitesimal generator (C-c.i.g.) of S(t) is the closure $\bar{G}$ of the "derivative" at $t = 0$ of $C^{-1}S(t)$. Representation theorems of S(t) in terms of $\bar{G}$ are proved. Also, necessary and sufficient conditions (generalizing those for (C_0)-semigroups) for a closed operator to be a C-c.i.g. are given. It is shown that, if the abstract Cauchy problem: $u'(t) = Au(t)$, $u(0) = x$, where A and C commute, has a unique solution with $\|u(t)\| \leq Me^{at}\|C^{-1}x\|$ for all $x \in CD(A)$, and if CD(A) is a core of A, then A is a C-c.i.g. (The converse of a result by Davies-Pang.) Finally, connections with semigroups of growth order α are established.

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