

**Le Bruyn, Lieven; Procesi, Claudio**

**Étale local structure of matrix invariants and concomitants.** (English) Zbl 0634.14034  
Algebraic groups, Proc. Symp., Utrecht/Neth. 1986, Lect. Notes Math. 1271, 143-175 (1987).

[For the entire collection see [Zbl 0619.00008](#).]

The authors study the variety  $V_{mn}$  built as an “approximation” of the space of orbits of  $X_{mn} = \oplus_{i=1}^m M_n(\mathbb{C})$  under action by componentwise conjugation of  $GL_n(\mathbb{C})$  by using  $GL_n(\mathbb{C})$ -invariants as parameters of  $V_{mn}$ . It parametrizes naturally the closed orbits of this action. The coordinate ring of  $V_{mn}$  is the so-called trace ring  $\pi_{mn}$  of  $m$  generic  $n \times n$ -matrices [see *C. Procesi*; Adv. Math. 19, 306-381 (1976; [Zbl 0331.15021](#))]. A point  $\xi$  of  $V_{mn}$  is of representation type  $\tau = (e_1, k_1; \dots; e_r, k_r)$  provided the corresponding isomorphism class of semisimple representations is built from  $r$  distinct simple components of dimensions  $k_i$  occurring with multiplicities  $e_i$ . It is shown that  $V_{mn}(\tau)$ , the subset of  $V_{mn}$  consisting of all points of representation type  $\tau$ , form a finite stratification into locally closed smooth subvarieties, where  $V_{mn}(\tau)$  lies in the closure of  $V_{mn}(\tau')$  if and only if  $\tau$  is a degeneration of  $\tau'$ . Furthermore, there is an explicit determination of the étale local structure of points of representation type with multiplicities 1. It is shown that this yields a Cohen-Macaulay module and its Poincaré series satisfies a certain functional equation.

These results are used in order to show that the singular locus of  $V_{mn}$  is determined by the Formanek center of  $\pi_{mn}$ . It concludes with the solution of the regularity problem for trace rings of generic matrices, i.e.,  $gl \dim(\pi_{mn}) < \infty$  if and only if  $m$  or  $n$  is equal to one or  $(m, n) = (2, 2), (2, 3)$  or  $(3, 2)$ . The authors use the work of *D. Luna* [Bull. Soc. Math. Fr., Suppl., Mém. No.33, 81-105 (1973; [Zbl 0286.14014](#))] and *R. P. Stanley* [Invent. Math. 68, 175- 193 (1982; [Zbl 0516.10009](#))].

Reviewer: [P.Schenzel](#)

**MSC:**

- [14M12](#) Determinantal varieties
- [20G05](#) Representation theory for linear algebraic groups
- [15A24](#) Matrix equations and identities
- [14L30](#) Group actions on varieties or schemes (quotients)

Cited in **1** Review  
Cited in **6** Documents

**Keywords:**

[general linear](#); [action by componentwise conjugation](#); [trace ring](#); [Formanek center](#)