Quefféléc, Martine
Substitution dynamical systems - spectral analysis. (English) [Zbl 0642.28013]
Lecture Notes in Mathematics, 1294. Berlin etc.: Springer-Verlag. XIII, 240 p.; DM 42.50 (1987).

The substitutions of constant length, also called uniform morphisms, are familiar to combinatoricians and theoretical computer scientists. The most famous example is the Thue-Morse sequence, defined as one of the fixed points of the substitution $0 \rightarrow 01$, $1 \rightarrow 10$, and beginning by 0110100110010110.... These substitutions have many number-theoretical properties [see *F. M. Dekking, M. Mendès France* and *A. J. van der Poorten*, Math. Intell. 4, 130-138 (1982; Zbl 0493.10001) and 4, 173-181 (1982; Zbl 0493.10002); see also the reviewer, Expo. Math. 5, 239-266 (1987)], but they can also be studied in terms of dynamical systems: the aim of this book is to give the spectral analysis of these dynamical systems.

The author first recalls the properties of the algebra of measures on the torus, then gives the spectral theory of unitary operators and of dynamical systems, such as dynamical systems associated to sequences. She then applies the results to the case of dynamical systems associated to sequences generated by constant length substitutions: this culminates with the complete description of the maximal spectral type of all admissible substitutions, using matrix Riesz products. The book is self-contained (for instance a proof of the Perron-Frobenius theorem is given), for which the reader is grateful, but sometimes lost if he does not read the book in a linear way.

This book is not only very useful but also exciting, giving links between combinatorics (or theoretical computer science), ergodic theory and the harmonic analysis of measures. (The reader may discover some misprints, for instance the first line of page 106 should be read p(n)$\leq C.n$).

Reviewer: J.-P.Allouche

MSC:

28D99	Measure-theoretic ergodic theory
11K55	Metric theory of other algorithms and expansions; measure and Hausdorff dimension
11-02	Research exposition (monographs, survey articles) pertaining to number theory
28-02	Research exposition (monographs, survey articles) pertaining to measure and integration

Cited in **7** Reviews
Cited in **202** Documents

Keywords:

finite automata; substitutions of constant length; uniform morphisms; Thue-Morse sequence; dynamical systems; spectral analysis; maximal spectral type; ergodic theory; harmonic analysis of measures