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Stability of vector bundles and extremal metrics. (English) Zbl 0645.53037

Invent. Math. 92, No. 2, 403-407 (1988).

The problem of finding Calabi extremal metrics on a compact Kähler manifold M depends on the existence of holomorphic vector fields on M and on the structure of its algebra. In the present paper negative examples are constructed. The authors take a complex surface $S_0 = C \times \mathbb{P}^1$, where C is a compact Riemann surface of genus $g \geq 2$, and the Kähler metric g_0 which is the product of the metric of constant curvature -1 on C and that of constant curvature +1 on $\mathbb{P}^1$. (This metric has scalar curvature $R \equiv 0$).

Writing S_0 in terms of vector bundles over C, namely $S_0 = \mathbb{P}(E_0)$, $E_0 = C \times \mathbb{C}^2$, the authors deform E_0 appropriately in order to construct new ruled surfaces S over C such that 1) S does not admit an extremal Kähler metric g whose Kähler class $= [\omega_0]$ in $H^2(S, \mathbb{R}) = H^2(S_0, \mathbb{R})$ (here ω_0 denotes the Kähler form of g_0 on S_0). 2) there are no non-trivial holomorphic vector fields on S. The found obstruction involves the borderline semi-stability properties of Hermitian vector bundles with Hermite-Einstein connections.

Reviewer: S.Dimiev

MSC:

53C55 Global differential geometry of Hermitian and Kählerian manifolds
32Q99 Complex manifolds

Keywords:

Kähler-Einstein metric; positive first Chern class; Calabi extremal metrics; negative examples; holomorphic vector fields

Full Text: DOI EuDML

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