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Generalized solution of the control optimization problem with a nonclassical functional.

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The paper deals with the control optimization problem for a dynamical process

$$(*) \quad \dot{x} = f(x, t) + \phi(x, t)u, \quad x \in E \quad n, \quad u \in E \quad r$$

with the optimal criterion

$$I = \int_{t_1}^{t_2} \{Q_g(x, \theta) + \sum_{j=1}^r k_j^{-1} [u \quad q_j(\theta) + p^{-1}u \quad q_{jopt}(\theta)]\} d\theta,$$

where u_{opt} is an unknown optimal control minimizing I on the solution of $(*)$, $k_j > 0$, $p^{-1} + q^{-1} = 1$, u q_j is an even function of u_j . It is shown that the functional I has a unique minimum for $u = u_{opt}$ defined by

$$(\partial V / \partial x) \phi(x, t) = -(\partial / \partial u) U_g(u_{opt}, t),$$

where $V = V(x, t)$ is the solution of the Lyapunov equation for the uncontrolled process $(*)$ ($u \equiv 0$). Moreover, the paper shows that the optimization of stochastic control with uncertainty concerning the observable noise leads to functionals in the form of conditional mathematical expectations which are used for control processes optimized in a deterministic way. The optimal (or suboptimal) control in both the deterministic and the stochastic case can be created via the solution of the Lyapunov equation under adequate boundary conditions. An algorithm is presented that generates this solution.

Reviewer: [W.Hejmo](#)

MSC:

- 49J15 Existence theories for optimal control problems involving ordinary differential equations
- 49M99 Numerical methods in optimal control
- 93C15 Control/observation systems governed by ordinary differential equations
- 49J55 Existence of optimal solutions to problems involving randomness
- 93E20 Optimal stochastic control

Keywords:

nonclassical optimal control; generalized solution; Lyapunov equation