Jannsen, Uwe
Continuous étale cohomology. (English) Zbl 0649.14011
Math. Ann. 280, No. 2, 207-245 (1988).

The author shows how to construct a very well-behaved p-adic cohomology theory, called continuous cohomology by deriving the left exact functor

$$\{\text{inverse system } (F_n) \text{ of étale sheaves on } X\} \to \text{ abelian groups } (F_n) \mapsto \varprojlim_n H_0(X, F_n).$$

The construction, when applied to locally constant sheaves, F_n, gives the continuous étale cohomology theory of *W. G. Dwyer* and *E. M. Friedlander* [Trans. Am. Math. Soc. 292, 247–280 (1985; Zbl 0581.14012)]. However, the author's construction applies to arbitrary sheaves while enjoying all the desirable properties of a cohomology theory (e.g. Hochschild-Serre spectral sequences, Chern classes, a Milnor $\varprojlim^1$ sequence to relate it to ℓ-adic cohomology). All in all, continuous cohomology looks to be one way around a number of technical difficulties in ℓ-adic cohomology.

Reviewer: V.P.Snaith

MSC:

14F20	Étale and other Grothendieck topologies and (co)homologies
14F30	p-adic cohomology, crystalline cohomology
18G10	Resolutions; derived functors (category-theoretic aspects)
14C35	Applications of methods of algebraic K-theory in algebraic geometry

Keywords:

p-adic cohomology theory; continuous cohomology

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