Miles, John

Resonance and symmetry breaking for the pendulum. (English) Zbl 0656.70024

Physica D 31, No. 2, 252-268 (1988).

Periodic solutions of the differential equation for periodic forcing of a lightly damped pendulum are obtained on the alternative hypotheses that (i) the contributions of second and higher harmonics to the average Lagrangian are negligible or (ii) the solution is close to that for free oscillations. The resonance curves (amplitude or root-mean energy vs. driving frequency) and stability boundaries for symmetric swinging oscillations and their asymmetric descendents (following symmetry breaking) are determined for $\delta \ll 1$ and $\epsilon = \mathcal{O}(\delta)$, where δ is the ratio of actual to critical damping, and ϵ is the ratio of the maximum external moment to the maximum gravitational moment. Resonance, as defined by synchronism between the external moment and the damping moment, is found to be impossible, and the conventional resonance curve separates into two branches, if $\epsilon > \epsilon_* = 3.28\delta + \mathcal{O}(\delta^3)$, which condition is necessary for normal symmetry breaking. A numerical, Fourier-series determination of the resonance curve and bifurcation points for $\delta = 1/8$ and $\epsilon = 1/2$ is presented in an appendix by *P. J. Bryant*.

MSC:

70J30 Free motions in linear vibration theory
37-XX Dynamical systems and ergodic theory
70J25 Stability for problems in linear vibration theory

Keywords:

Periodic solutions; periodic forcing; lightly damped pendulum; average Lagrangian; resonance curves; stability boundaries; symmetric swinging oscillations; synchronism; Fourier-series determination; bifurcation points

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