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Relaxation of a variational method for impedance computed tomography. (English)

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The problem studied in the paper is the following: find the electrical conductivity of a body by means of voltage and current flux measurements at the boundary. Mathematically, the problem can be studied by means of an elliptic partial differential equation

$$(1)\quad div(\gamma(x)Du)=0 \quad in \quad \Omega\subset\mathbb{R}^n \quad (n\geq 2)$$

where u is the voltage, γ the unknown conductivity, and γ Du the current flux. The boundary measurements are given by the map $\Lambda_\gamma : H^{1/2}(\partial\Omega)\to H^{-1/2}(\partial\Omega)$ which maps a function $\phi\in H^{1/2}(\partial\Omega)$ into $\gamma Du_\phi\nu\in H^{-1/2}(\partial\Omega)$, where μ_ϕ is the solution of (1) with $u=\phi$ on $\partial\Omega$.

The problem is then reduced to the minimization for a functional of the form (2) $\int_\Omega g(DU)dx$ $(U=U_0$ on $\partial\Omega)$ where U is a vector-valued function and g is Borel measurable. By using the fact that the relaxed problem associated to (2) is $\int_\Omega Qg(DU)dx$, where Qg denotes the quasiconvex envelope of g, the authors deduce a relaxed formulation for the impedance computed tomography problem, which seems to be more efficient than the original one from the point of view of practical calculations.

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MSC:

49J45 Methods involving semicontinuity and convergence; relaxation
35J20 Variational methods for second-order elliptic equations
78A55 Technical applications of optics and electromagnetic theory
35R30 Inverse problems for PDEs

Keywords:
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