Donnelly, Harold; Fefferman, Charles
Nodal sets of eigenfunctions on Riemannian manifolds. (English) Zbl 0659.58047
Invent. Math. 93, No. 1, 161-183 (1988).

Let Δ denote the Laplacian of a compact connected Riemannian manifold M. Suppose that F is a real eigenfunction of Δ with eigenvalue λ. It is proved that F vanishes to at most order $c\sqrt{\lambda}$, for any point in M. The nodal set N of F is defined to be the set of points where F vanishes. If M is real analytic, upper and lower bounds are obtained for the n-1-dimensional Hausdorff measure of N. More specifically, $c_1\sqrt{\lambda} \leq \mathcal{H}^{n-1}N \leq c_2\sqrt{\lambda}$.

Reviewer: H.Donnelly

MSC:

58J50 Spectral problems; spectral geometry; scattering theory on manifolds
53C20 Global Riemannian geometry, including pinching

Keywords:

Laplacian; eigenfunction; nodal set

Full Text: DOI EuDML

References:

[1] Aronszajn, N.: A unique continuation theorem for solutions of elliptic partial differential equations of second order. J. Math. Pures Appl.36, 235-249 (1957) · Zbl 0084.30402

[2] Bishop, R., Crittenden, R.: Geometry of Manifolds. New York-London: Academic Press, 1964 · Zbl 0132.16003

[3] Brüning, J.: Über Knoten von Eigenfunktionen des Laplace Beltrami operator. Math. Z.158, 15-21 (1978) · Zbl 0358.58015 · doi:10.1007/BF01214561

[4] Cheeger, J., Gromov, M., Taylor, M.: Finite propagation speed, kernel estimates for functions of the Laplace operator, and the geometry of complete Riemannian manifolds. J. Differ. Geom.17, 15-53 (1982) · Zbl 0493.53035

[5] Cheng, S. Y.: Eigenfunctions and nodal sets. Comment. Math. Helv.51, 43-55 (1976) · Zbl 0334.35022 · doi:10.1007/BF02568142

[6] Cheng, S. Y., Yau, S. T.: Differential equations on Riemannian manifolds and their geometric applications. Commun. Pure Appl. Math.28, 333-354 (1975) · Zbl 0312.53031 · doi:10.1002/cpa.3160280303

[7] Federer, H.: Geometric measure, theory. Berlin-Heidelberg-New York: Springer 1969 · Zbl 0176.00801

[8] Hörmander, L.: Linear partial differential operators. Berlin-Heidelberg-New York: Springer 1963 · Zbl 0108.09301

[9] Stein, E., Weiss, G.: Fourier analysis on Euclidean spaces. Princeton: Princeton University Press 1971 · Zbl 0232.42007