Rauch, Jeffrey; **Reed, Michael C.**

Nonlinear superposition and absorption of delta waves in one space dimension. (English)

Zbl 0661.35058
J. Funct. Anal. 73, 152-178 (1987).

The author deals with problems of the following types: let u^ϵ be the solution of the semilinear strictly hyperbolic system

$$(*) \quad (\partial_t + A(x,t)\partial_x + B(x,t))u = f(x,t,u)$$

with the initial data of the form $g + h^\epsilon$, where g is "classical" and smooth h^ϵ converge to a distribution μ. Then u^ϵ converge to $\bar{u} + \sigma$ in specified sense, where $\bar{u}$ is a solution of (*), $\bar{u}(t=0) = g$, and σ is a solution of (*) with $f = 0$, $\sigma(t=0) = \mu$. This fact expresses a superposition principle: the singular part of the solution propagates linearly, the classical part propagates by the nonlinear equation. The distribution μ is a singular measure for f sublinear or can be more singular for f bounded. If f satisfies condition of superlinear dissipation $\lim_{|u_j|\to\infty} f_j(x,t,u)/u_j = -\infty$ and $sgn(u_j)f_j(x,t,u) \le c(l + \sum |u_i|)$ (the solution do not blow up in finite time), then $u^\epsilon \to \bar{u}$, i.e. the singular part is absorbed.

Reviewer: A.Doktor

MSC:

35L60	First-order nonlinear hyperbolic equations
35L45	Initial value problems for first-order hyperbolic systems
35B30	Dependence of solutions to PDEs on initial and/or boundary data and/or on parameters of PDEs
35B65	Smoothness and regularity of solutions to PDEs
35B40	Asymptotic behavior of solutions to PDEs

Keywords:

distribution initial data; semilinear strictly hyperbolic; classical; smooth; superposition principle; singular part; singular measure; superlinear dissipation

Full Text: DOI

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