Yaméogo, Joachim

Sur l'alignement dans les schémas de Hilbert ponctuels du plan. La famille des N-uplets de $\mathbb{P}^2$ contenant au moins r points sur une droite. (Alignment in the punctual Hilbert schemes of the plane. The family of N-tuples containing at least r points on a line). (French)

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Let $Hilb^N\mathbb{P}^2$ be the Hilbert scheme parametrizing the closed finite subschemes Z of length N in the projective plane. In this paper we are interested in the stratification of these schemes Z by the number of points they have on a line. The subject arises from a study made by *J. Brun* and *André Hirschowitz* on the stratification of $Hilb^N\mathbb{P}^2$ by "postulation". The methods and techniques we use are those developed by *A. Iarrobino*, *J. Briançon* and *M. Granger*, who studied the geometry of $Hilb^N\mathbb{C}\{x, y\}$. There are essentially three steps which could be summarized as follows:

First, consider the subschemes Z which are supported on one point and contain a fixed subscheme S of length r: we obtain a scheme of dimension $N - r$ and list its irreducible components.

Fix a line L and consider those Z which have at least r points on L: we prove that the corresonding scheme is irreducible of dimension $2N - r$.

Fix only N and r (r at least 2): we prove that the corresponding Hilbert scheme is irreducible of dimension $2N - r$.

Reviewer: J.Yameogo

MSC:

14C05 Parametrization (Chow and Hilbert schemes)
14N05 Projective techniques in algebraic geometry
32S60 Stratifications; constructible sheaves; intersection cohomology (complex-analytic aspects)
14D99 Families, fibrations in algebraic geometry

Keywords:

vertical-escalier; Zariski tangent space; Hilbert scheme; stratification; postulation

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