Terwilliger, Paul

A characterization of P- and Q-polynomial association schemes. (English) Zbl 0663.05016

J. Comb. Theory, Ser. A 45, 8-26 (1987).

Let $Y=(X,(R_i),\ 0\leq i\leq d)$ be a symmetric d-class association scheme, with intersection numbers p^h_{ij} and Krein parameters q^h_{ij} ($0\leq h,i,j\leq d$), and for each i ($0\leq i\leq d$) define the ith (reduced) intersection diagram D_i (resp. representation diagram D^*_i) on the nodes $0,1,...,d$ drawing an undirected arc between any distinct h, j for which $p^h_{ij}>0$ (resp. $q^h_{ij}>0$). Y is called P-polynomial (resp. Q-polynomial) if some D_i (resp. D^*_i) is a path.We obtain pointwise semi-definite matrices $G(i)$ and $G(i)^*$ ($0\leq i\leq d$) that yield new inequalities for the p^h_{ij} and q^h_{ij}. We show for each i ($0\leq i\leq d$), D^*_i being a forest, the vanishing of $G(i)$, and the existence of a certain geometric representation of X are all equivalent. A similar result relates $G(i)^*$ and D_i. Denoting by a leaf in any diagram a node adjacent to exactly one other, we show there is at most one leaf besides the O-node in any connected D^*_i for a P-polynomial scheme. We combine this with the above results and get an interpretation of the Q-polynomial property for P-polynomial schemes. Finally, we use equations induced by the vanishing of some $G(i)$ to obtain a simple proof of a theorem of D. Leonard, that the intersection numbers of a P- and Q-polynomial scheme can be found from 5 parameters.

Reviewer: Reviewer (Berlin)

MSC:

05B30 Other designs, configurations

Keywords:

symmetric d-class association scheme; pointwise semi-definite matrices; geometric representation; leaf; Q-polynomial property; P-polynomial schemes

Full Text: DOI

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