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Deformation spaces associated to compact hyperbolic manifolds. (English) Zbl 0664.53023

Discrete groups in geometry and analysis, Pap. Hon. G. D. Mostow 60th Birthday, Prog. Math. 67, 48-106 (1987).

[For the entire collection see Zbl 0632.00015.]

The paper deals with the geometry of group representations, namely most of it is devoted to the study of the spaces C(M) and P(M) of marked conformal and projective structures on a hyperbolic closed n-manifold $M = H^n/G$, $\pi_1(M) = G \subset IsomH^n$. Of course, if $n \geq 3$ then the Mostow Rigidity Theorem states that the space of hyperbolic structures on M is a point. The paper's main theme is that this is far from being true for C(M) and P(M). [The first result in this direction was given by the reviewer's curves in C(M) - see Ann. Math. Stud. 97, 21- 31 (1981; Zbl 0464.30037)].

The paper studies some subspaces $C_B \subset C(M)$ and $P_B \subset P(M)$ corresponding to tending deformations of M along totally geodesic hypersurfaces $S \subset M$ [see the reviewer's curves mentioned above, and *D. Sullivan*'s generalization of Thurston's Mickey Mouse example - Bull. Am. Math. Soc., New Ser. 6, 57-73 (1982; Zbl 0489.58027)]. The first main result is that a lower bound for dim C_B and dim P_B (an algebraic approach) is given by the largest number of disjoint totally geodesic hypersurfaces $S \subset M$. Independent and different geometric approaches for this bound are contained in the reviewer's paper in Complex Analysis and Applications 1985, 14-28 (1986; Zbl 0624.30045) and a paper by *C.Konrouniotis* [Math. Proc. Camb. Philos. Soc. 98, 247-261 (1985; Zbl 0577.53041)].

The second main result of the paper is the discovery of non-isolated singularities (in irreducible points, too) of C(M) and P(M) for some M which are locally homeomorphic to certain singular algebraic varieties. Note that $C(M) \neq C_B$ in general and, moreover, this space C(M) is non-connected for certain manifolds M [see the reviewer, Deformations of conormal structures and Lobachevsky Geometry, Preprint MSRI, Berkeley, 1989.]

Reviewer: B.Apanasov

MSC:

53C30	Differential geometry of homogeneous manifolds
57M05	Fundamental group, presentations, free differential calculus
58H15	Deformations of general structures on manifolds

Cited in **4** Reviews
Cited in **38** Documents

Keywords:

hyperbolic manifold; conformal structures; group representations; projective structures; hyperbolic structures; tending deformations; totally geodesic hypersurfaces; singularities

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