Giaquinta, Mariano

The problem of the regularity of minimizers. (English) Zbl 0667.49029

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[For the entire collection see Zbl 0657.00005.]

This paper gives a survey of the regularity results for minimizers of variational integrals obtained by various authors during the past decades concentrating on the direct approach to regularity working directly with the functional instead of working with its Euler-Lagrange equations. Section 1 is devoted to a general existence theorem for quasiconvex integrals and contains a list of counterexamples to everywhere regularity in the vector-valued case. Section 2 introduces the notion of quasi- minima as an unifying concept. Besides other things one finds the important theorem that a reverse Hölder inequality holds for the gradient of a quasi-minimizer. Higher integrability of the gradient is the first step for developing a C^1-partial regularity theory for minima of uniform quasi-convex integrals which is discussed in section 3: imposing natural growth conditions on the integrand f and assuming that f is strictly uniform quasi-convex any local minimizer is of class $C^{1,\alpha}$ apart a closed (singular) set of vanishing Lebesgue measure. A final section improves this theorem for special classes of integrals giving more precise estimates on the size of the singular set. It is worth remarking that the paper contains a large number of references including the most important recent contributions to the subject.

Reviewer: M.Fuchs

MSC:

49Q20 Variational problems in a geometric measure-theoretic setting
49-02 Research exposition (monographs, survey articles) pertaining to calculus of variations and optimal control

Keywords:

survey; minimizers of variational integrals; quasiconvex integrals; quasi-minima; reverse Hölder inequality