Gyöngy, I.

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The stochastic abstract partial differential equation

$$(0)\quad du(t)=(A(t,\omega)u(t)+f(t,\omega))dV(t)+\sum_{i=1}^{d_1}(B_i(t,\omega)u(t)+g_i(t,\omega))\circ dM^i(t),\quad u(0)=u_0,$$

driven by a continuous increasing process V and continuous semimartingales M^i is approximated as $\delta \to 0$ by (abstract) partial differential equations

$$(\delta)\quad du_\delta(t)=(A_\delta(t,\omega)u_\delta(t)+f_\delta(t,\omega))dV_\delta(t)+\sum_{i=1}^{d_1}(B_{\delta i}(t,\omega)u_\delta(t)+g_{\delta_i}(t,\omega))dM^i_\delta(t),\quad u_\delta(0)=u_{\delta 0},$$

driven by continuous, adapted increasing processes V_δ and continuous, adapted processes M^i_δ of locally bounded variation. $((B_iu(t)+g_i(t))\circ M^i(t)$ denote certain stochastic differentials corresponding to the Stratonovich differentials).

In the special case when $A_\delta=A$, $f_\delta=f=0$, $B_{\delta i}=B_i$, $g_{\delta i}=g_i=0$ for every $\delta>0$, $i=1,\dots,d_1$, and $B_i's$ are time-independent nonrandom linear operators, the main result reads as follows:

Let $H_3\hookrightarrow H_2\hookrightarrow H_1\hookrightarrow H_0$ be a chain of embedded Hilbert spaces with continuous and dense injections. Assume

$$|Au|_{\alpha-2}\le K|u|_\alpha,\quad |B_iu|_{\beta-1}\le K|u|_\beta,\quad |B_i^*u|_1\le K|u|_2,$$

$$(1)\quad |[B_i,B_j]u|_{\alpha-1}\le K|u|_\alpha,\quad |(B_iu,u)|\le K|u|_0^2,\quad |(B_iu,B_ju)+(B_iB_ju,u)|\le K|u|_0^2,$$

$$|(B_iAu,u)+(Au,B_iu)|\le K|u|_1^2,\quad (u,Au)+\epsilon|u|_1^2\le K|u|_0^2,$$

for every $u\in H_3$, $t\in[0,T]$, $\omega\in\Omega$, $\alpha:=1,2,3$, $\beta:=0,1,2,3$, where $\epsilon>0$, $K\ge 0$ are constants, (,) denotes the scalar product in H_0, $|u|_\alpha$ is the norm of u in H_α, $[B_i,B_j]:=B_iB_j-B_jB_i$ and B_i^* is the adjoint of B_i with respect to the scalar in H_0;

$$(2)\quad V_\delta(t)\to V(t),\quad M^i_\delta(t)\to M^i(t),\quad S^{ij}_\delta(t)\to 0$$

as $\delta\to 0$, in probability uniformly in t on bounded intervals for every $i,j:=1,\dots,d_1$, where $S^{ij}_\delta(t):=\int_0^t(M^i-M^i_\delta)dM^j_\delta(s)-2^{-1}<M^i,M^j>(t)$.

The total variation of $S^{ij}_\delta s$ over the interval [0,T] is bounded uniformly in δ ;

$$(3)\quad |u_0-u_{\delta 0}|_0\to 0\quad in\quad probability.$$

Let u_δ and u be solutions of equations (δ) and (0), respectively, in the normal triplet $H_1\hookrightarrow H_0\equiv H_0^*\hookrightarrow H_1^*$ such that

$$\int_0^T|u_\delta(t)|_2^2dV_\delta(t)<\infty,\quad \int_0^T|u(t)|_3^2dV(t)<\infty,$$

$u_\delta\in C([0,T];H_1)$, $u\in C([0,T];H_2)$. Then under the above conditions: $\sup_{t\in[0,T]}|u_\delta(t)-u(t)|_0\to 0$ and $\int_0^T|u_\delta(t)-u(t)|_1^2dV_\delta(t)\to 0$ in probability.

[For part II, see the following review, Zbl 0669.60059].

Reviewer: I.Gyöngy

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