Gyöngy, I.

On the approximation of stochastic partial differential equations. II. (English) Zbl 0669.60059

Stochastics Stochastics Rep. 26, No. 3, 129-164 (1989).

[For part I see the preceding review, Zbl 0669.60058].

In this second part of our study we apply the results of the first part to the stochastic partial differential equation

$$(0)\quad du(t,\omega,x)=\sum_{p,q=0}^{d}D_p(a^{pq}D_qu(t,\omega,x)+f(t,\omega,x)dt+$$

$$+\sum_{p=0}^{d}\sum_{i=1}^{d_1}(b_i^pD_pu(t,\omega,x)+g_i(t,\omega,x))\circ dM^i(t),\quad u(0,\omega,x)=u_0(\omega,x),$$

approximated by the partial differential equations

$$(\delta)\quad du_\delta(t,\omega,x)=\sum_{p,q=0}^{d}D_p(a_\delta^{pq}D_qu_\delta(t,\omega,x)+f_\delta(t,\omega,x))dt+$$

$$+\sum_{p=0}^{d}\sum_{i=1}^{d_1}(b_{\delta i}^pD_pu_\delta(t,\omega,x)+g_{\delta i}(t,\omega,x))dM_\delta^i(t),\quad u_\delta(0,\omega,x)=u_{\delta 0}(\omega,x),$$

as $\delta\to 0$, where $D_k:=\partial/\partial x^k$ for $k:=1,...,d$ and D_0 is the identity. As an application we treat the problem of robustness in nonlinear filtering of diffusion processes. In the special case when $a_\delta^{pq}=a^{pq}$, $f_\delta=f=0$, $b_{\delta i}^p=b_i^p$, $g_{\delta i}=g_i=0$ and b_i^p are nonrandom time-independent, the main result can be formulated as follows:

Assume:

(1) The derivates in $x\in\mathbb{R}^d$ of the coefficients a^{pq}, b_i^p up to the second order are bounded measurable functions (well-measurable in t, ω), and the $d\times d$-matrix $(a^{k\ell})$ is uniformly elliptic.

(2) M_δ^i is an adapted absolutely continuous process for every δ, i, such that $M_\delta^i(t)\to M^i(t)$, $S_\delta^{ij}(t)\to 2^{-1}<M^i,M^j>(t)$ in probability, uniformly in $t\in[0,T]$, and the total variations of S_δ^{ij} over [0,T] are bounded in probability, where $S_\delta^{ij}(t):=\int_0^t(M^i-M_\delta^i)dM_\delta^j(s)$. The processes M_δ^i are continuous semimartingales whose bounded variation and quadratic variation have bounded derivatives in $t\in[0,T]$.

(3) $u_{\delta 0}$ and u_0 are $\mathcal{F}_0$-measurable random elements in $W_2^1(\mathbb{R}^d)$ and in $W_2^2(\mathbb{R}^d)$, respectively, such that $|u_{\delta 0}-u_0|_0\to 0$ in probability, where $|\cdot|_0$ denotes the norm in $L_2(\mathbb{R}^d)$.

Then under the above conditions equations (0) and (δ) admit a unique generalized solution u and u_δ, respectively, and

$$\sup|u_\delta(t)-u(t)|_0\to 0,\quad \int_0^T|u_\delta(t)-u(t)|_1^2dt\to 0\quad in\quad probability,$$

as $\delta\to 0$, where $|\cdot|_1$ denotes the norm in $W_2^1(\mathbb{R}^d)$.

Reviewer: I.Gyöngy

MSC:

60H15 Stochastic partial differential equations (aspects of stochastic analysis)

Keywords:

Sobolev spaces; problem of robustness; nonlinear filtering of diffusion processes; semimartingales

Full Text: DOI