Miyazaki, Mitsuhiro
Characterizations of Buchsbaum complexes. (English) Zbl 0671.13014
Manuscr. Math. 63, No. 2, 245-254 (1989).

Let K be a field, Δ a simplicial complex with vertex set $V \subset \{x_1, ..., x_n\}$, K[$\Delta$] the associated Stanley-Reisner ring, $A = k[x_1, ..., x_n]$ and $\mathfrak{m}_j = (x_1^j, ..., x_n^j)$. The author computes the modules $Ext_A^i(A/\mathfrak{m}_j, K[\Delta])$ in terms of the reduced simplicial cohomology of certain subcomplexes of Δ. As a corollary he gets Hochster's fundamental theorem which relates the local cohomology of K[Δ] with respect to $\mathfrak{m}$ and the reduced simplicial cohomology. After having recalled some criteria of Schenzel and Stückrad-Vogel for K[Δ] to be a Buchsbaum ring, the author proves his characterization of the Buchsbaum property of Stanley-Reisner rings:

K[Δ] is Buchsbaum if and only if for all $i < d$ the modules $Ext_A^i(A/\mathfrak{m}, K[\Delta])$ and $Ext_A^i(A/\mathfrak{m}_2, K[\Delta])$ have the same length.

Reviewer: W.Bruns

MSC:

13H10 Special types (Cohen-Macaulay, Gorenstein, Buchsbaum, etc.)
13D03 (Co)homology of commutative rings and algebras (e.g., Hochschild, André-Quillen, cyclic, dihedral, etc.)
55U10 Simplicial sets and complexes in algebraic topology

Keywords:

Stanley-Reisner ring; Ext; Buchsbaum ring

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