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Extensions of results of Komlós, Major, and Tusnády to the multivariate case. (English)

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The purpose of this excellent paper is to establish extensions of the well-known Komlós-Major-Tusnády strong approximations to the multivariate case. One of the main results can be stated as follows:

Let H be a continuous, nonnegative function on [0,∞) such that $H(t)/t^{3+r}$ is eventually increasing for some $r > 0$ and log H(t)/t$^{1/2}$ is eventually non-increasing. Suppose X is a d- dimensional random variable with mean 0, covariance matrix Σ and E H($|X|$) $< \infty$, where $|\cdot|$ denotes the Euclidean norm. Then i.i.d. sequences $\{X_n\}$, $\{Y_n\}$ can be constructed in such a way that $X_n =^D X$, Y_n is N(0,Σ)- distributed and

$$T_n = \sum_1^n X_k - \sum_1^n Y_k = O(H^{-1}(n)) \quad a.s.$$

If the moment generating function of X exists and satisfies a mild smoothness condition then the rate O(log n) can be achieved. Simultaneously with the above, anologues of KMT-type inequalities (e.g. exponential inequalities) for T_n are also obtained.

The basic tool in the proofs is an extension of the quantile transformation method of KMT to the multidimensional case. To get this a large deviation theorem for conditional distribution functions is first proved.

Reviewer: T.Inglot

MSC:

60F15 Strong limit theorems
60F17 Functional limit theorems; invariance principles
60F10 Large deviations

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multivariate quantile transformation; strong approximations; KMT-type inequalities; large deviation theorem; conditional distribution

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