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Turbulence and the dynamics of coherent structures I: Coherent structures. II: Symmetries and transformations. III: Dynamics and scaling. (English) Zbl 0676.76047

Q. Appl. Math. 45, 561-590 (1987).

Two separate developments in recent years have altered the basic statistical framework of turbulence established by *G. I. Taylor* [Transition and turbulence. Proc. Roy. Soc. A 151, 421-478 (1938). From the laboratory there is abundant information implying the existence of coherent structures and revealing something of their nature. On the theoretical side recent applications of dynamical systems theory to turbulence suggest that such flows reside on relatively low-dimensional manifolds or attractors. However, in the first instance, no general framework incorporating structures into turbulence theory has emerged. In the second instance, direct means have not been put forward for the description of these attractors.

The present papers present a program for dealing with both of these issues in a unified manner. An essential ingredient of the treatment given here is the basic idea by *J. L. Lumley* [Proc. Symp., Madison/USA 1980, 215-242 (1981; Zbl 0486.76073)] that spatial velocity correlations be orthogonally decomposed as a rational and quantitative method of identifying coherent structures. The use of this procedure has been hampered by the lack of complete and sufficiently resolved data. Present day experimental techniques and numerical data have greatly remedied this problem. However, due to the laborious nature of the method it has remained unsuitable for dealing with the large data sets which have become available. As a result reduction to a one-dimensional calculation is usually forced. The methods presented in Pt. I overcome this shortcoming and making fully three-dimensional flows accessible to treatment.

The orthogonal decomposition of the covariance is a classical result which is referred to as the Karhunen-Loeve expansion in pattern recognition and as factor or principal-component analysis in the statistical literature Lumley refers to it as the proper orthogonal expansion. In Pt. I we review and further develop this procedure within the context of fluid mechanics. We then apply this to the point of obtaining practical methods for the determination of coherent structures of a turbulent flow. Since the Karhunen-Loeve expansion is known to be optimal this has significance for data compression. Although this aspect of the method is not pursued here, we mention that compressions of $O(10^3)$ have been indicated by the method thus far.

Both the significance and application of coherent structures to a dynamical description of turbulence is presented in Pt. III. This is relevant to the widely held view that chaotic, dissipative, dynamical systems are eventually drawn into a strange attractor which is of relatively low dimension. Indeed a number of fluid experiments support this view. Theoretical estimates also support this view but greatly overestimate the dimension and alas provide no clue to the parametrization of the attractor. While we do not confront this issue directly it will be seen that a practical, and in a sense optimal, description of the attrator is furnished. The procedure developed here does give an upper bound on the dimension of the attractor through the actual construction of an embedding space. The work presented here does not represent a new theory of turbulence in the sense for example of closure schemes. Rather it is a methodology which in a practical sense is without approximation and which can be applied to a wide variety of turbulent flows of current interest. This methodology depends in an essential way on the availability of sufficient turbulence data, numerical or experimental, in basic geometries, Since it is often the case that the data set is insufficient for an accurate determination of the coherent structures, we explore, in Pt. II, the use of symmetries to extend the available data.

As will be seen, in certain key cases, orders of magnitude increases in data can be extracted from existing data bases through the use of invariance groups for a flow geometry. Also in Pt. II we consider the transformation of coherent structures for use in other geometries. Under transformation the property of being a coherent structure is lost, but the resulting set of functions nevertheless provide a useful basis set for related geometries. In a similar vein in Pt. III we deal with the alteration of coherent structures under changes of parameter, such as Reynolds number, Rayleigh number, and so forth. Again, while the variation does not leave invariant the property of being a coherent structure, it still provides a useful functional basis.

MSC:

76F99 Turbulence

37D45 Strange attractors, chaotic dynamics of systems with hyperbolic behavior

Keywords:

coherent structures; low-dimensional manifolds; attractors; fully three- dimensional flows; orthogonal decomposition; Karhunen-Loeve expansion; pattern recognition; principal-component analysis; proper orthogonal expansion; strange attractor; closure schemes

Full Text: DOI