Kottwitz, Robert E.
Tamagawa numbers. (English) Zbl 0678.22012
Ann. Math. (2) 127, No. 3, 629-646 (1988).

The main theorem is that the Tamagawa number $\tau(G)$ of a connected reductive algebraic group G defined over a number field F is invariant under passage from G to an inner twist. Since it is known that $\tau(G) = 1$ if G is simply-connected, semisimple and quasisplit over F this theorem completes the proof of the Weil conjecture: $\tau(G) = 1$ for G simply-connected, semisimple. A key ingredient in the proof is the p-adic Euler-Poincaré function introduced here. The orbital integrals of this function are shown to have remarkable properties. In particular, the function may be used in Arthur's simple trace formula and the formula then reduces to its stable part (this requires the Hasse principle). The stable parts for G and an inner twist G' are compared and the formula $\tau(G) = \tau(G')$ is squeezed out in familiar fashion. The Euler- Poincaré function is also shown to give a quick proof of Rogawski's theorem on the Shalika germ for the identity element of a p-adic group. Finally, a theorem attributed to Casselman says that the Euler-Poincaré function is, up to a given constant, a pseudo-coefficient of the Steinberg representation.

Reviewer: D. Shelstad

MSC:

22E55 Representations of Lie and linear algebraic groups over global fields and adèle rings
11F70 Representation-theoretic methods; automorphic representations over local and global fields
12G05 Galois cohomology

Keywords:

Tamagawa number; connected reductive algebraic group; inner twist; Weil conjecture; p-adic Euler-Poincaré function; orbital integrals; simple trace formula; Hasse principle; Shalika germ; p-adic group; Steinberg representation

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