

**Smith, Dean E.**

**On the Cohen-Macaulay property in commutative algebra and simplicial topology.** (English)

Zbl 0686.13008

Pac. J. Math. 141, No. 1, 165-196 (1990).

A ring  $R$  is called a “ring of sections” provided  $R$  is the section ring of a sheaf  $(\mathcal{A}, X)$  of commutative rings defined over a base space  $X$  which is a finite partially ordered set given the order topology. Regard  $X$  as a finite abstract complex, where a chain in  $X$  corresponds to a simplex. In specific instances of  $(\mathcal{A}, X)$ , certain algebraic invariants of  $R$  are equivalent to certain topological invariants of  $X$ . (Author)

The author investigates the depth of factor rings of  $SR(F, \Sigma)$ , the Stanley-Reisner ring of a complex  $\Sigma$  with coefficients in a field  $F$ .  $SR(F, \Sigma)$  is viewed as the ring of sections of a sheaf of polynomial rings over the partially ordered set of all simplices of  $\Sigma$ . The complex  $\Sigma$  is defined to be Cohen-Macaulay (CM) provided the reduced singular cohomology of the link subcomplexes vanish except in maximal degree. The main theorem goes as follows: Let  $S$  be the polynomial ring  $S = F[X_0, \dots, X_n]$ , put  $\alpha = n - pd_S SR(F, \Sigma)$ , then the skeleton  $\Sigma^\alpha$  is maximal with respect to the property of being CM.

Reviewer: [Y.Felix](#)

**MSC:**

- 13H10 Special types (Cohen-Macaulay, Gorenstein, Buchsbaum, etc.)
- 55U05 Abstract complexes in algebraic topology
- 13D25 Complexes (MSC2000)
- 55M99 Classical topics in algebraic topology
- 57Q99 PL-topology
- 18F20 Presheaves and sheaves, stacks, descent conditions (category-theoretic aspects)
- 13F20 Polynomial rings and ideals; rings of integer-valued polynomials

Cited in **2** Reviews  
Cited in **13** Documents

**Keywords:**

section ring of a sheaf of commutative rings; ring of sections; finite partially ordered set; order topology; finite abstract complex; simplicial complex; Stanley-Reisner ring of a complex; sections of a sheaf of polynomial rings; Cohen-Macaulay

**Full Text:** [DOI](#)