Shibata, Yoshihiro; Kikuchi, Masahiro

On the mixed problem for some quasilinear hyperbolic system with fully nonlinear boundary condition. (English) Zbl 0689.35055

J. Differ. Equations 80, No. 1, 154-197 (1989).

The authors prove the local existence theorem in time of classical solutions to the following mixed problem for second-order systems:

$$(N)\quad \partial_t^2 u(t) - \partial_i(p^i(t, D'u(t)) + \phi_\Omega(t, D'u(t)) = f_\Omega(t) \quad in \quad (0,T)\times\Omega$$

$\nu_i p^i(t, D'u(t)) + Q_r(t, D'u(t)) = f_r(t)$ on $(0,T)\times\Gamma$

u(0)= u_0 and $\partial_t u(0) = u_1$ in Ω

here u denotes an m-vector.

(N) was already treated and the local existence theorem was proved by Y. Shibata and G. Nakamura. But the order of Sobolev spaces in which solutions exist was not best possible. T. Kato treated the mixed problems of the same type as in (N) in his abstract framework. When $m = 1$ and the nonlinear function p^i, Q_Ω and Q_Γ do not depend on t and $\partial_t u$, and $f_\Gamma(t) \in 0$; applying his abstract theory to (N), he gave some improvements of the result due to Y. Shibata regarding the minimal order of the Sobolev spaces in which the solution exists. The purpose of this paper is to give same improvements as the ones given by Kato, where $m \geq 1$, where the nonlinear functions may depend on t and $\partial_t u$ and $f_\Gamma(t) \not\equiv 0$. The approach used in this paper is concrete and elementary, and different from the one used by Kato.

Reviewer: J.Wang

MSC:

35L70 Second-order nonlinear hyperbolic equations
35L55 Higher-order hyperbolic systems
35A07 Local existence and uniqueness theorems (PDE) (MSC2000)

Keywords:

fully nonlinear boundary condition; quasilinear system; mixed problem

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