Le Bruyn, Lieven; **Procesi, Claudio**
Semisimple representations of quivers. (English) Zbl 0693.16018
Trans. Am. Math. Soc. 317, No. 2, 585-598 (1990).

Let Q be a finite quiver with vertices $Q_0 = \{1, ..., n\}$ and let us fix an algebraically closed field C of characteristic zero and a dimension vector $\alpha = (\alpha(i))_{i \in Q_0}$. In the sense of *P. Gabriel* [Manuscr. Math. 6, 71-103 (1972; Zbl 0232.08001)], the set of C-representations of Q with dimension vector α, R(Q,α), is an affine variety where the linear reductive group $GL(\alpha) = \prod_i GL_{\alpha(i)}(C)$ acts by isomorphisms of the category of representations.

The question which is considered here is to study the orbit structure of GL(α) acting on R(Q,α). A representation V in R(Q,α) is called semisimple (resp. nilpotent) if its orbit GL(α)$\cdot V$ is closed (resp. if 0 belongs to the Zariski closure of GL(α)$\cdot V$). Every representation V has a Jordan decomposition $V = V_s + V_n$, where V_s is semisimple and V_n is nilpotent. One of the main objectives of the paper is to study the semisimple representations of Q by applying the étale slice machinery devised by *D. Luna* [in Bull. Soc. Math. Fr., Mém. 33, 81-105 (1973; Zbl 0286.14014)]. One of the byproducts is the determination of all dimension vectors which correspond to a semisimple representation of Q.

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MSC:

16G20 Representations of quivers and partially ordered sets
14L30 Group actions on varieties or schemes (quotients)
20G05 Representation theory for linear algebraic groups

Keywords:

finite quiver; dimension vector; affine variety; linear reductive group; category of representations; Jordan decomposition; semisimple representations

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