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A note on some inequalities for means. (English) [Zbl 0693.26005](#)
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The logarithmic and identric means of two positive numbers a and b are defined by $L = L(a, b) := (b - a)/(\ln b - \ln a)$ for $a \neq b$; $L(a, a) = a$, and $I = I(a, b) := \frac{1}{e}(b^b/a^a)^{1/(b-a)}$ for $a \neq b$, $I(a, a) = a$, respectively. Let $A = A(a, b) := (a + b)/2$ and $G = G(a, b) := \sqrt{ab}$ denote the arithmetic and geometric means of a and b , respectively. Recently, in two interesting papers, H. Alzer has obtained the following inequalities: (1) $A \cdot G < L \cdot I$ and $L + I < A + G$; (2) $\sqrt{G \cdot I} < L < \frac{1}{2}(G + I)$ which hold true for all positive $a \neq b$. In our paper we prove, by using new methods, that the left side of (1) is weaker than the left side of (2), while the right side of (1) is stronger than the right side of (2).

Reviewer: [J.Sándor](#)

MSC:

26D15 Inequalities for sums, series and integrals

Cited in **2** Reviews
Cited in **30** Documents

Keywords:

logarithmic mean; arithmetic mean; geometric mean; identric mean; inequalities

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