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On quasi-Hadamard products of p-valent functions with negative coefficients. (English)
Zbl 0693.30015

Let $S_0(k, p, \alpha)$ denote the class of analytic and p-valent functions of the form
\[ f(z) = a_p z^p - \sum_{n=1}^{\infty} a_{p+n} z^{p+n}; \quad a_p > 0, \quad a_{p+n} \geq 0 \quad \text{for} \quad p \in \mathbb{N} \]
and such that
\[ \sum_{n=1}^{\infty} \frac{p+n}{p} k(p+n-\alpha) a_{p+n} \leq (p-\alpha) a_p, \]
where $\alpha \in \mathbb{R}$, $p \in \mathbb{N}$, $k \geq 0$. We define the quasi-Hadamard product of the functions
\[ f(z) = a_p z^p - \sum_{n=1}^{\infty} a_{p+n} z^{p+n}; \quad g(z) = b_p z^p - \sum_{n=1}^{\infty} b_{p+n} z^{p+n} \]
as follows
\[ f \ast g(z) := a_p b_p z^p - \sum_{n=1}^{\infty} a_{p+n} b_{p+n} z^{p+n}. \]

In this paper some properties of the quasi-Hadamard product are given. In particular it is proved: If
\[ f_i(z) \in S_0(0, p, \alpha_j), \quad j = 1, 2, \ldots, m \]
then $f_1 \ast f_2 \ast \cdots \ast f_m(z) \in S_0(m-1, p, \beta)$ where $\beta = \max\{\alpha_1, \alpha_2, \ldots, \alpha_m\}$.

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MSC:
\begin{align*}
30C45 & \text{Special classes of univalent and multivalent functions of one complex variable (starlike, convex, bounded rotation, etc.)} \\
30C75 & \text{Extremal problems for conformal and quasiconformal mappings, other methods}
\end{align*}

Keywords:
p-valent functions; quasi-Hadamard product