Pheidas, Thanases

Hilbert's tenth problem for fields of rational functions over finite fields. (English)

Zbl 0696.12022
Invent. Math. 103, No. 1, 1-8 (1991).

Let F be a finite field of characteristic $p > 2$ and F(t) the field of rational functions in the variable t with coefficients in F. It is proved that the existential theory of F(t) in the language $\{+, \cdot, 0, 1, t\}$ is undecidable, and so, there is no algorithm to solve arbitrary polynomial equations over F(t); this is an analogue to Hilbert's tenth problem for the case of a function field. Due to the similarities between $\mathbb{Q}$ (the field of rationals) and the common properties of function fields over finite fields, when it comes to the solvability of diophantine equations, this result seems to suggest that the analogue of Hilbert's tenth problem for $\mathbb{Q}$ has a negative answer.

Reviewer: T.Pheidas

MSC:

11U05 Decidability (number-theoretic aspects)
11R58 Arithmetic theory of algebraic function fields
11D99 Diophantine equations

Keywords:

polynomial equations over field of rational functions; Hilbert's tenth problem; function fields over finite fields

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paper as accurately as possible without claiming the completeness or perfect precision of the matching.