Petunin, Yu. I.; **Semejko, N. G.**

The stochastic process of segments on the two-dimensional Euclidean sphere. II. (Russian)

Zbl 0696.60016
Teor. Veroyatn. Mat. Stat., Kiev 41, 88-96 (1989).

This work continues the authors' researches beginning in ibid. 39, 107- 113 (1988; Zbl 0665.60019), and devoted to the further study of the properties of the sphere covered by random caps [see *R. E. Miles*, Sankhyā, Ser. A 33, 145-174 (1971; Zbl 0243.60014); *P. A. P. Moran* and *S. Fazekas de St. Groth*, Biometrika 49, 389-396 (1962; Zbl 0108.316)].

We investigate a random cap process $\mathcal{A}$ on the two-dimensional Euclidean sphere S^2 of unit radius. Each trajectory of the process $\mathcal{A}$ is an unordered set $\{Q_{(i)}(U_{(i)}, a_{(i)})\}$ consisting of mutually disjoint hemispherical caps. A position of any cap $Q_{(i)}(U_{(i)}, a_{(i)})$ on S^2 is identically determined by the pair $[U_{(i)}(\phi_{(i)}, \theta_{(i)}); a_{(i)}]$ where $(\phi_{(i)}, \theta_{(i)})$ are the spherical coordinates of the random cap center $U_{(i)}$. Cap diameters $\{a_{(i)}\}$ take values from the general population $K = [0, A]$ where $A < \pi$ with probability density f(a) and N is a nonnegative integer random variable. The process $\mathcal{A}$ is considered as a random unordered marked point process (MPP) $(\mathcal{E}^*_{\mathcal{A}}, \mathcal{X}^*_{\mathcal{A}}, P^*_{\mathcal{A}})$ with the trajectories $E^*_{\mathcal{A}} = \{[U_{(i)}; a_{(i)}]\} \in \mathcal{E}^*_{\mathcal{A}}$ in the bounded space (Y, U_y, B_y) where $Y = S^2 \times K$, $U_y = U_{S^2} \otimes U_k$, $B_y = B_{S^2} \odot B_k$ [see the authors, loc. cit.]. We can put in correspondence a random unordered MPP of parameters $\mathcal{D} = (\mathcal{E}^*_{\mathcal{D}}, \mathcal{X}^*_{\mathcal{D}}, P^*_{\mathcal{D}})$ with the trajectories $E^*_{\mathcal{D}} = \{[\phi_{(i)}, \theta_{(i)}; a_{(i)}]\}$ in the bounded space (Z, U_z, B_z) where $Z = \Delta_{\phi,\theta} \times K$ ($\Delta_{\phi,\theta} = \{(\phi, \theta) : 0 \le \phi < 2\pi,\ -\pi/2 < \theta < \pi/2\} \cup \{(0, \pi/2), (0, -\pi/2)\}$) to the process $\mathcal{A}$.

We shall propose that the processes $\mathcal{A}$ and $\mathcal{D}$ possess the following properties:

1. The random variable N has a finite expectation: $E[N] < \infty$.

2. The point process (PP) $\tilde{\mathcal{A}} = (\mathcal{E}_{\tilde{\mathcal{A}}}, U_{\tilde{\mathcal{A}}}, P_{\tilde{\mathcal{A}}})$ of the random cap centers has a constant intensity λ.

3. The MPP $\mathcal{D}$ is a random unordered simple PP with independent marking in the bounded space (Z, U_z, B_z) [see the authors, loc. cit.].

Theorem. For any $\bar{Z} \in U_z$ a moment measure of the first order $O^{(1)}(\bar{Z}) = E[N^*(E^*_D, \bar{Z})]$ of the process $\mathcal{D}$ is calculated by the formula

$$O^{(1)}(\bar{Z}) = \iiint_{(\phi,\theta,a)\in\bar{Z}} \lambda \cos\theta \quad f(a) d\phi d\theta da$$

where $N^*(E^*_{\mathcal{D}}, \bar{Z}) = card[E^*_{\mathcal{D}} \cap \bar{Z}]$.

Reviewer: Yu.I.Petunin

MSC:

60D05 Geometric probability and stochastic geometry
60G55 Point processes (e.g., Poisson, Cox, Hawkes processes)

Keywords:

properties of the sphere covered by random caps; random cap process; point process