

**Casselman, W.**

**Canonical extensions of Harish-Chandra modules to representations of  $G$ .** (English)

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Can. J. Math. 41, No. 3, 385-438 (1989).

Let  $G$  be the group of  $\mathbb{R}$ -rational points on a connected reductive algebraic group defined over  $\mathbb{R}$ . Let  $K$  be a maximal compact subgroup and  $\mathfrak{g}$  the complexified Lie algebra of  $G$ . Assume  $G$  to be algebraically embedded as a closed subset of a finite-dimensional matrix algebra, and let  $g \mapsto \|g\|$  be the associated norm. A continuous representation  $\pi$  of  $G$  on a topological vector space  $V$  is called of moderate growth if  $V$  is a Fréchet space and for every continuous semi-norm  $\rho$  on  $V$  there exists a positive integer  $N$  and a continuous semi-norm  $\nu$  such that  $\rho(\pi(g)v) \leq \|g\|^N \nu(v)$  for all  $g \in G$  and  $v \in V$ .

The author proves that to each finitely generated Harish-Chandra module  $V$  one can attach (up to canonical topological isomorphism) a unique smooth representation of  $G$  of moderate growth whose underlying  $(\mathfrak{g}, K)$ -module is isomorphic to  $V$ . This assignment is an exact functor from the category of finitely generated Harish-Chandra modules into the category of smooth representations of moderate growth. As an application, he obtains results on asymptotic behavior of (not necessarily  $K$ -finite) matrix coefficients of smooth representations of moderate growth.

These results are in part joint work with Nolan Wallach, and this paper is essentially a sequel to *N. Wallach* [in Lie Group Representations I, Lect. Notes Math. 1024, 287–369 (1983; Zbl 0553.22005)].

Reviewer: [D. Miličić](#)

**MSC:**

[22E46](#) Semisimple Lie groups and their representations

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connected reductive algebraic group; maximal compact subgroup; complexified Lie algebra; finitely generated Harish-Chandra module; smooth representations of moderate growth

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