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On the ℓ-adic cohomology of varieties over number fields and its Galois cohomology. (English)

Zbl 0703.14010
Galois groups over $\mathbb{Q}$, Proc. Workshop, Berkeley/CA (USA) 1987, Publ., Math. Sci. Res. Inst. 16, 315-359 (1989).

[For the entire collection see Zbl 0684.00005.]

Let k be a number field. The Galois group $G_k = Gal(\bar{k}/k)$ acts on the étale cohomology groups $H^i(X_{\bar{k}}, \mathbb{Q}_\ell/\mathbb{Z}_\ell(n))$, for any smooth projective variety X defined over k. One can thus study the continuous cohomology $H^\nu(G_{\bar{k}}, H^i(X_{\bar{k}}, \mathbb{Q}_\ell/\mathbb{Z}_\ell(n)))$. The paper under review consists of a survey about what is known about these groups, especially for the case $\nu = 2$. In addition some interesting conjectures are stated and discussed, examples are given, and function field analogues are proven. An important tool in studying these groups is of course Deligne's proof of the Weil conjectures.

Although many relations with e.g. algebraic K-theory, the Beilinson conjectures, Iwasawa theory and the study of algebraic cycles amply suggest the importance of these groups, yet it seems that not much has been proven about them in general so far.

Reviewer: J.Top

MSC:

14F20	Étale and other Grothendieck topologies and (co)homologies
11R34	Galois cohomology
14F30	p-adic cohomology, crystalline cohomology
14C35	Applications of methods of algebraic K-theory in algebraic geometry
11R70	K-theory of global fields

Keywords:

ℓ-adic cohomology; Galois cohomology; Deligne cohomology; Weil conjectures; Beilinson conjectures