Lions, P. L.
On the Schwarz alternating method. III: A variant for nonoverlapping subdomains. (English)
Zbl 0704.65090
Domain decomposition methods for partial differential equations, Proc. 3rd Int. Symp. Houston/TX (USA) 1989, 202-223 (1990).

[For the entire collection see Zbl 0695.00026.]

This paper is the third part of a series of three articles by the author [part I: ibid., 1st Int. Symp., Paris 1987, 1-42 (1989; Zbl 0658.65090); Part II: ibid., 2nd Int. Symp., Los Angeles 1988, 47-70 (1989; Zbl 0681.65072)] on the Schwartz alternating method for solving partial differential equations. In this part, the iteration is based on a nonoverlapping decomposition of the domain $\Omega = \Omega_1 \cup ... \cup \Omega_m \cup \Sigma$ into m subdomains Ω_i, where $\Sigma = \cup \gamma_{ij}$ and $\gamma_{ij} = \partial \Omega_i \cap \partial \Omega_j \setminus \partial \Omega$ with $i \neq j$.

For the model problem $-\Delta u = f$ in Ω and $u = 0$ on $\partial \Omega$, the iteration sequence $\{(u_i^n)_{i=1,...,m}\}_{n=0,1,...}$ consists of m functions $u_i^n(x)$ defined on $\bar{\Omega}_i$, where, for given $(u_j^n)_{j=1,...,m'}$ u_i^{n+1} will be defined as the solution of the partial differential $-\Delta u_i^{n+1} = f$ in Ω_i under Fourier's boundary conditions

$$\partial u_i^{n+1}/\partial n_{ij} + \lambda_{ij} u_i^{n+1} = \partial u_j^n/\partial n_{ij} + \lambda_{ij} u_j^n \quad (\lambda_{ij} = \lambda_{ji} > 0)$$

on the interface γ_{ij} (j$\neq i$) and under the given boundary condition $u_i^{n+1} = 0$ on $\partial \Omega_i \cap \partial \Omega$. Here n_{ij} is the unit outward normal to $\partial \Omega_i$ on γ_{ij}. The author shows weak convergence of u_i^n to $u|_{\Omega_i}$ in $H^1(\Omega_i)$ and of $u_i^n|_{\gamma_{ij}}$ to $u|_{\gamma_{ij}}$ in $H^{1/2}(\gamma_{ij})$. Extensions to more complicated problems including convection-diffusion problems are given.

Reviewer: U.Langer

MSC:

65N55	Multigrid methods; domain decomposition for boundary value problems involving PDEs
65N12	Stability and convergence of numerical methods for boundary value problems involving PDEs
65N22	Numerical solution of discretized equations for boundary value problems involving PDEs
65F10	Iterative numerical methods for linear systems
35J05	Laplace operator, Helmholtz equation (reduced wave equation), Poisson equation

Cited in **10** Reviews
Cited in **76** Documents

Keywords:

domain decomposition methods; Poisson equation; nonoverlapping subdomains; Schwartz alternating method; convergence; convection- diffusion problems