Axelsson, O.; **Layton, W.**

Defect correction methods for convection dominated convection-diffusion problems. (English)

Zbl 0705.65081
RAIRO, Modélisation Math. Anal. Numér. 24, No. 4, 423-455 (1990).

The authors consider the approximate solution of singularly perturbed convection diffusion equations. Any appropriate numerical method faces the problem of resolution of sharp boundary layers. For instance, standard finite element methods are not appropriate when the perturbation parameter ϵ is of smaller order than the mesh distance h.

The method studied herein, originally due to *P. W. Hemker* [Lect. Notes Math. 960, 485-501 (1982; Zbl 0505.65047)], is a combination of defect correction with an artificial viscosity approximation. It computes a sequence of finite element solutions, where for $\epsilon \ll h$ the first solution is only a first order accurate approximation. At each correction step the residual is computed and a correction to the current approximation is calculated using the first order viscosity approximation.

The authors give local and global error estimates and show how the convergence depends on solution regularity, types of layers present and the subdomains on which the error is measured.

Reviewer: E.Lanckau

MSC:

65N30 Finite element, Rayleigh-Ritz and Galerkin methods for boundary value problems involving PDEs

65N15 Error bounds for boundary value problems involving PDEs

35J70 Degenerate elliptic equations

Keywords:

singularly perturbed convection diffusion equations; sharp boundary layers; finite element methods; defect correction; artificial viscosity; local and global error estimates; convergence

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