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Convergence to equilibrium in large time for Boltzmann and B.G.K. equations. (English)

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Summary: Rarefied gas dynamics is usually described by the Boltzmann equation: $\partial_t f + v \cdot \nabla_x f = Q(f, f)$, where f(t,x,v) is the density of particles which at time t and point x move with velocity v, and Q is a quadratic collision term described e.g. by *C. Cercignani* [The Boltzmann equation and its applications (1988; Zbl 0646.76001)]. A large amount of information on the behavior of the gas is contained in the simpler model of B.G.K.: $\partial_t f + v \cdot \nabla_x f = M_f - f$, where M_f is the Maxwellian having the same moments as f. From the physical point of view, the density of particles is assumed to converge to an equilibrium represented by a Maxwellian function of the velocity v when the time t becomes large. This Maxwellian is assumed to be constant when x lies in a bounded domain with suitable boundary conditions or in a periodic box. In that case, the walls are said to "thermalize" the gas.

The goal of this work is to give some mathematical results on these topics. Note that this problem has already been studied in the case of the spatially homogeneous Boltzmann equation and in the case of the full Boltzmann equation in a periodic box. We systematically use the results on existence of solutions to the Boltzmann equation stated by *R. J. DiPerna* and *P.-L. Lions* in [Ann. Math., II. Ser. 130, No.2, 321-366 (1989; Zbl 0698.45010)] and their extension when f is assumed to satisfy various boundary conditions obtained by *K. Hamdache* [Global existence of weak solutions for the initial boundary value problems of the Boltzmann equation (to appear)]. In Section 2 we establish mathematically the convergence of f to a Maxwellian satisfying a free transport equation when x varies in a bounded domain. A complete study of Maxwellians which satisfy this condition is given in Section 3. Section 4 is devoted to using this description together with boundary conditions in order to establish the thermalizing effect of the walls. The speed of convergence to equilibrium is studied in Section 5. And finally, Section 6 is devoted to the study of strong convergence.

MSC:

76P05	Rarefied gas flows, Boltzmann equation in fluid mechanics
82B40	Kinetic theory of gases in equilibrium statistical mechanics
82D05	Statistical mechanical studies of gases
82C40	Kinetic theory of gases in time-dependent statistical mechanics

Keywords:

Boltzmann equation; quadratic collision term; existence of solutions; convergence; free transport equation; boundary conditions; convergence to equilibrium

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