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On directed groups. (English. Russian original) Zbl 0714.06007

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A directed set $(M; \leq)$ can be transformed into an algebra $(M; \sigma)$ of signature $\sigma = \{\vee, \wedge\}$ which contains all binary operations \vee and \wedge defined in the following way: if $x, y \in M$, $x \leq y$ we set $x \vee y = y$ and $x \wedge y = x$. If x and y are uncomparable, we choose (and fix) for $x \vee y$ an upper bound of x and y . Dually for $x \wedge y$. The operations can be defined as commutative ones. 1.2 Theorem. Let $(M; \leq)$ be a directed set and $(M; \sigma)$ an algebra of signature $\sigma = \{\vee, \wedge\}$ which is constructed by means of \leq in the described way. Then there holds in $(M; \sigma)$ H1) $x \vee x = x$, $x \wedge x = x$, H2) $x \vee y = y \vee x$, $x \wedge y = y \wedge x$, H3) $x \vee (x \vee y) = x \vee y$, $x \wedge (x \wedge y) = x \wedge y$, H4) $(x \vee y) \vee z = x \vee ((x \vee y) \vee z)$ and dually, H5) $x \vee (x \wedge y) = x$ and dually. Conversely, if an order relation \leq is introduced in an algebra $(M; \sigma)$ so that $x \leq y$ iff $x \vee y = y$ (or $x \wedge y = x$) then $(M; \leq)$ is a directed set. If $(G; \cdot, e, ^{-1})$ is a group and $(G; \leq)$ a directed set, then $(G; \cdot, e, ^{-1}, \leq)$ is called a directed group if $x \leq y$ implies $zxt \leq zyt$ for arbitrary $z, t \in G$. 2.2 Theorem. If $(G; \cdot, e, ^{-1}, \leq)$ is a directed group then there can be defined operations \vee and \wedge in G so that H1- H5 hold and moreover H6) $zxt \vee z(x \vee y)t = z(x \vee y)t$ and dually, and H7) $(x \vee y)^{-1} = x^{-1} \vee y^{-1}$ and dually. Conversely, every algebra $(G; \cdot, e, ^{-1}, \vee, \wedge)$ so that $(G; \cdot, e, ^{-1})$ is a group which fulfils H1-H7, turns out to be a directed group if we put $x \leq y$ iff $x \vee y = y$. Other properties of directed groups are deduced which are near to the ℓ -group properties.

Reviewer: F.Šik

MSC:

06F15 Ordered groups
20F60 Ordered groups (group-theoretic aspects)

Cited in **2** Reviews
Cited in **2** Documents

Keywords:

homogeneous directed group; d-group; d-homomorphism; l-group; directed set

Full Text: [DOI](#)

References:

- [1] L. Fuks, Partially Ordered Algebraic Systems [in Russian], Nauka, Moscow (1965).
- [2] A. I. Mal'tsev, Algebraic Systems [in Russian], Nauka, Moscow (1970).

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